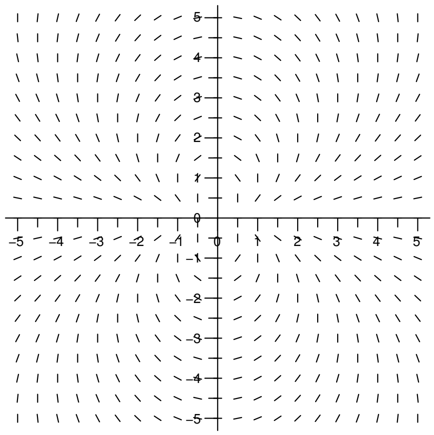
2.1.1(4). $f(x+iy) = x^2 - y^2 + 2ixy = u + iv$ where $u = x^2 - y^2$ and $v = 2xy$. This function (vector field) is defined on $D = \mathbb{C}$.

We compute $\dfrac{\partial u}{\partial x} = 2x$, $\dfrac{\partial u}{\partial y} = -2y$, $\dfrac{\partial v}{\partial x} = 2y$, $\dfrac{\partial v}{\partial y} = 2x$.

Since $\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 4y \neq 0$, f is not locally (or globally) irrotational.

Since $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 4x \neq 0$, f is not locally (or globally) sourceless. (Thanks to Tony Donadio for the correction.)

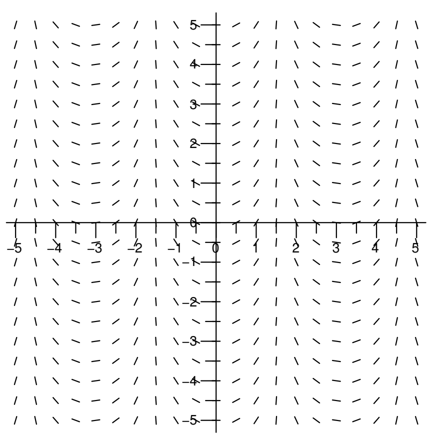
The streamlines of f are circles centered in ir with radius $|r|$, $r \in \mathbb{R}$. To see this, you can check that $\gamma(t) = ir + |r|e^{it}$ satisfies $\frac{f(\gamma(t))}{\gamma'(t)} = \frac{\gamma(t)^2}{\gamma'(t)} \in \mathbb{R}$ for every $t \in [0, 2\pi]$.



2.1.1(6). $f(x+iy) = e^y \cos(x) + ie^y \sin(x)$ is defined on $D = \mathbb{C}$. Notice that

$$f(z) = \overline{\exp(-iz)}$$

is the complex conjugate of an analytic function. This implies that f is locally irrotational and sourceless. Since $D = \mathbb{C}$ is simply-connected, f is also globally irrotational and sourceless.



2.2(2). $a_k = \dfrac{(k!)^2}{(2k)!}$.

We have $|a_{k+1}/a_k| = \dfrac{((k+1)!)^2(2k)!}{(2k+2)!(k!)^2} = \dfrac{(k+1)^2}{(2k+1)(2k+2)} \to 1/4$ for $k \to \infty$.
So the radius of convergence is $R = 4$.

2.2(10). $\dfrac{1+z}{1-z} = (1+z)\dfrac{1}{1-z} = (1+z)\displaystyle\sum_{k=0}^{\infty} z^k = \sum_{k=0}^{\infty}(z^k + z^{k+1}) = 1 + 2\sum_{k=1}^{\infty} z^k$.

2.2(14). $\displaystyle\sum_{k=0}^{\infty} \frac{z^{2n}}{n!} = \sum_{k=0}^{\infty} \frac{(z^2)^n}{n!} = \exp(z^2)$.

2.2(18). Define $f(z) = \dfrac{1}{1-z} = \displaystyle\sum_{n=0}^{\infty} z^n$.

Then $f'(z) = \dfrac{1}{(1-z)^2} = \displaystyle\sum_{n=1}^{\infty} nz^{n-1}$ and $f''(z) = \dfrac{2}{(1-z)^3} = \displaystyle\sum_{n=2}^{\infty} n(n-1)z^{n-2}$.

It follows that $\dfrac{2z^2}{(1-z)^3} = \displaystyle\sum_{n=2}^{\infty} n(n-1)z^n$, since both sides are equal to $z^2 f''(z)$.

2.2(22). (a) $f(z) = \displaystyle\sum_{n=0}^{\infty} a_n(z-z_0)^n$ for $|z-z_0| < R$. If $f(z) = 0$ for all z with $|z-z_0| < r$, where $0 < r < R$, then $a_n = \dfrac{f^{(n)}(z_0)}{n!} = 0$ for each n.

(b) If $F(z) = \sum a_n(z-z_0)^n$ and $G(z) = \sum b_n(z-z_0)^n$ are equal for all z with $|z-z_0| < r$, $r > 0$, then $F(z) - G(z) = \sum(a_n - b_n)(z-z_0)^n = 0$ for $|z-z_0| < r$, so $a_n - b_n = 0$ for each n.