5.1: 7, 8,9, 10, 11, 19, 23, 26, 40, 41, 43, 44, 46

7 $\int \frac{dx}{2x} = \frac{1}{2}\int \frac{dx}{x} = \frac{1}{2}\ln(x) + C.$

8 $\int 14e^x dx = 14\int e^x dx = 14e^x + C.$

9 $\int (6u^2 - 3\cos(u))du = 2\int 3u^2 du - 3\int \cos(u)du = 2u^3 - 3\sin(u) + C.$

10 $\int (5t^3 - \sqrt{t})dt = 5\int t^3 dt - \int t^{1/2}dt = 5\frac{t^4}{4} - \frac{2t^{3/2}}{3} + C.$

11 $\int sec^2(\theta)d\theta = \tan(\theta) + C.$

19 $\int x(x + \sqrt{x})dx = \int (x^2 + x^{3/2})dx = \int x^2 dx + \int x^{3/2}dx = \frac{x^3}{3} + \frac{2x^{5/2}}{5} + C.$

23 $\int (2x^2 + 5)^2 dx = \int (4x^3 + 20x^2 + 25)dx = \int 4x^3 dx + 20\int x^2 dx + 25\int 1dx = x^4 + \frac{20x^3}{3} + 25x + C.$

26

$$\int \frac{x^2 + \sqrt{x} + 1}{x^2}dx = \int (1 + x^{-3/2} + x^{-2})dx = \int 1dx + \int x^{-3/2}dx + \int x^{-2}dx$$
$$= x - 2x^{-1/2} - x^{-1} + C = x - \frac{2}{\sqrt{x}} - \frac{1}{x}$$

40 A ball is thrown directly upward from ground level with an initial velocity of 96 ft/s. Assuming that the ball's only acceleration is that due to gravity (that is, $a(t) = -32$ ft/s^2), determine the maximum height reached by the ball and the time it takes to return to ground level.

We know that velocity is the antiderivative of acceleration, so $v(t) = \int a(t)dt = \int -32dt = -32t + C$. To find C, we use the information that the initial velocity, $v(0) = 96$, so $96 = v(0) = -32 * 0 + C = C$, so $v(t) = -32t + 96$. We also know that height is the antiderivative of velocity, so $h(t) = \int v(t)dt = \int (-32t + 96)dt = -32\int tdt + 96\int dt = -16t^2 + 96t + D$ where D is some constant. We know that the ball started from the ground, so $h(0) = 0$, so $D = 0$ and $h(t) = -16t^2 + 96t$. To find the maximum height, we find to find when the derivative of height (velocity) is zero. This occurs when $-32t + 96 = 0$ or $t = 3$. We know this is a maximum because $h'(t) = v(t) > 0$ for $t < 3$ and $h'(t) = v(t) < 0$ for $t > 3$.

Therefore, the maximum height is $h(3) = -16 * 9 + 96 * 3 = 144ft$. To find when the ball hits the ground again, we set $h(t) = 0 = -16t^2 + 96t = t(-16t + 96)$, so the ball hits the ground the second time when $-16t + 96 = 0$, or when $t = 6$.

41 The marginal cost of a certain commodity is $C'(x) = 6x^2 - 2x + 5$, where x is the level of production. If it costs \$5 to produce 1 unit, what is the total cost of producing 5 units.

We know that cost is the antiderivative of marginal cost, so $C(x) = \int C'(x)dx = \int (6x^2 - 2x + 5)dx = 2\int 3x^2 dx - \int 2x dx + 5\int dx = 2x^3 - x^2 + 5x + D$. Since it costs \$5 to produce 1 unit, $C(1) = 5$, so $C(1) = 2 - 1 + 5 + D = 6 + D = 5$, so $D = -1$ and $C(x) = 2x^3 - x^2 + 5x - 1$. Finally the cost of producing 5 units is $C(5) = 2 * 125 - 25 + 25 - 1 = \$249$.

43 It is estimated that t months from now, the population of a certain town will be changing at a rate of $4+5t^{2/3}$ people per month. If the current population is $10,000$, what will the population be 8 months from now?

Let $P(t)$ be the population t months from now. Then $P'(t) = 4 + 5t^{2/3}$ and $P(t) = \int (4 + 5t^{2/3})dt = 4\int dt + 5\int t^{2/3}dt = 4t + 3t^{5/3} + C$. The current population being $10,000$ means that $P(0) = 10,000$, so $C = 10,000$ and $P(t) = 4t + 3t^{5/3} + 10,000$. Then the population 8 months from now will be $P(8) = 32 + 3 * 32 + 10,000 = 10,128$ people.

44 A particle travels along the x-axis in such a way that its acceleration at time t is $a(t) = \sqrt{t} + t^2$. If it starts at the origin with an initial velocity of 2 (that is, $s(0) = 0$ and $v(0) = 2$), determine its position and velocity when $t = 4$.

$v(t) = \int a(t)dt = \int (\sqrt{t} + t^2)dt = \frac{2t^{3/2}}{3} + \frac{t^3}{3} + C$, $2 = v(0) = C$, so $v(t) = \frac{2t^{3/2}}{3} + \frac{t^3}{3} + 2$. Then $s(t) = \int v(t)dt = \int \left(\frac{2t^{3/2}}{3} + \frac{t^3}{3} + 2\right) dt = \frac{4t^{5/2}}{15} + \frac{t^4}{12} + 2t + D$. Since $D = s(0) = 0$, $s(t) = \frac{4t^{5/2}}{15} + \frac{t^4}{12} + 2t$. Therefore, $v(4) = \frac{2*4^{3/2}}{3} + \frac{4^3}{3} + 2 = \frac{16}{3} + \frac{64}{3} + \frac{6}{3} = \frac{86}{3} \approx 28.67$ and $s(4) = \frac{4*4^{5/2}}{15} + \frac{4^4}{12} + 2 * 4 = \frac{568}{15} \approx 37.87$.

46 The price of bacon is currently \$1.80/lb in Styxville. A consumer service has conducted a study predicting that t months from now, the price will be changing at the rate of $0.084 + 0.012\sqrt{t}$ cents per month. How much will a pound of bacon cost 4 months from now?

Let $P(t)$ be the price of a pound of bacon t months from now. Then $P'(t) = 0.084 + 0.012\sqrt{t}$, so $P(t) = \int P'(t)dt = \int (0.084 + 0.012\sqrt{t})dt = 0.084t + 0.008t^{3/2} + C$. Since the current price is \$1.80/lb, then $1.8 = P(0) = C$, so $P(t) = 0.084t + 0.008t^{3/2} + 1.8$, so the price 4 months from now will be $P(4) = 0.084 * 4 + 0.008 * 4^{3/2} + 1.8 = \$2.2/\text{lb}$.