

5.4: 2,7,10,15, 16, 26, 29, 33, 36, 39, 44, 49, 54

2.

$$\int_{-5}^7 (-3)dx = -3x \Big|_{-5}^7 = -21 - (-3)(-5) = -36$$

7.

$$\int_1^2 \frac{c}{x^3} dx = \frac{-1}{2} \frac{c}{x^2} \Big|_1^2 = \frac{-1}{2} \left[\frac{c}{4} - c \right] = \frac{3c}{8}$$

10.

$$\int_0^2 7x^{1/3} dx = \frac{3x^{4/3}}{4} \Big|_0^2 = \frac{3}{4} [81 - 0] = \frac{243}{4}$$

15.

$$\int_{\ln 2}^{\ln 5} 5e^x dx = 5e^x \Big|_{\ln 2}^{\ln 5} = 5 [e^{\ln 5} - e^{\ln 2}] = 5(5 - 2) = 15$$

16.

$$\int_{e^{-2}}^e \frac{dx}{x} = \ln(x) \Big|_{e^{-2}}^e = \ln(e) - \ln(e^{-2}) = 1 + 2 = 3$$

26.

$$\int_1^2 \frac{x^3 + 1}{x} dx = \int_1^2 x^2 + \frac{1}{x} = \left(\frac{x^3}{3} + \ln(x) \right) \Big|_1^2 = \left(\frac{8}{3} + \ln(2) \right) - \left(\frac{1}{3} + \ln(1) \right) = \frac{7}{3} + \ln(2)$$

29.

$$\begin{aligned} \int_{-1}^2 (x + |x|) dx &= \int_{-1}^0 (x + |x|) dx + \int_0^2 (x + |x|) dx = \int_{-1}^0 (x - x) dx + \int_0^2 (x + x) dx \\ &= \int_0^2 2x dx = x^2 \Big|_0^2 = 4 - 0 = 4 \end{aligned}$$

In problems 31-38, find the area of the region under the given curve over the prescribed interval.

33. $y = \sec^2 x$ on $[0, \pi/4]$.

$$A = \int_0^{\pi/4} \sec^2(x) dx = \tan x \Big|_0^{\pi/4} = \tan(\pi/4) - \tan(0) = 1 - 0 = 1$$

36. $y = (x^2 + x + 1)\sqrt{x}$ on $[1, 4]$

$$\begin{aligned} A &= \int_1^4 (x^2 + x + 1)\sqrt{x} dx = \int_1^4 (x^{5/2} + x^{3/2} + x^{1/2}) dx \\ &= \frac{2x^{7/2}}{7} + \frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3} \Big|_1^4 = \frac{5602}{105} \approx 53.35 \end{aligned}$$

In problems 39-44, find the derivative of the given function.

39. $F(x) = \int_0^x \frac{t^2-1}{\sqrt{t+1}} dt$

By the second fundamental theorem of calculus,

$$F'(x) = \frac{x^2 - 1}{\sqrt{x + 1}}$$

44. $F(x) = \int_{\pi/3}^x \sec^2(t) \tan t dt.$

By the second fundamental theorem of calculus,

$$F'(x) = \sec^2(x) \tan(x)$$

The formula is taken from a table of integrals. Use differentiation to verify that the formula is correct.

49.

$$\begin{aligned} \int \frac{u du}{\sqrt{a^2 - u^2}} &= -\sqrt{a^2 - u^2} + C \\ \frac{d}{du} \left[-\sqrt{a^2 - u^2} + C \right] &= -\frac{1}{2}(a^2 - u^2)^{-1/2}(-2u) = \frac{u}{\sqrt{a^2 - u^2}} \end{aligned}$$

54. Evaluate

$$\int_0^2 f(x) dx \text{ where } f(x) = \begin{cases} x^3 & \text{if } 0 \leq x < 1 \\ x^4 & \text{if } 1 \leq x \leq 2 \end{cases}$$

The function is continuous so the integral exists.

$$\int_0^2 f(x) dx = \int_0^1 x^3 dx + \int_1^2 x^4 dx = \frac{x^4}{4} \Big|_0^1 + \frac{x^5}{5} \Big|_1^2 = \left(\frac{1}{4} - 0 \right) + \left(\frac{32}{5} - \frac{1}{5} \right) = 6.45$$

5.5: 1,3,6,9,10,15,17, 18, 23, 29, 32, 42, 46,51, 53

1a $\int_0^4 (2t + 4) dt = t^2 + 4t \Big|_0^4 = 32$

1b $\int_0^4 (2t + 4)^{-1/2} dt, u = 2t + 4, du = 2dt, dt = \frac{1}{2} du. \int_0^4 (2t + 4)^{-1/2} dt = \frac{1}{2} \int_4^{12} u^{-1/2} du = \frac{1}{2} 2u^{1/2} \Big|_4^{12} = u^{1/2} \Big|_4^{12} = 2\sqrt{3} - 2.$

3a $\int_0^\pi \cos t dt = \sin t|_0^\pi = \sin(\pi) - \sin(0) = 0 - 0 = 0.$

3b $\int_0^\pi t \cos(t^2) dt, u = t^2, du = 2t dt, t dt = \frac{1}{2} du. \int_0^\pi t \cos(t^2) dt = \int_0^{2\pi} \frac{1}{2} \cos(u) du = \frac{1}{2} \sin(u)|_0^{2\pi} = \frac{1}{2} (\sin(2\pi) - \sin(0)) = \frac{1}{2}(0 - 0) = 0.$

6a $\int x(3x^2 - 5) dx = \int (3x^3 - 5x) dx = \frac{3x^4}{4} - \frac{5x^2}{2} + C.$

6b $\int x(3x^2 - 5)^{50} dx, u = 3x^2 - 5, du = 6x dx, x dx = \frac{1}{6} du. \int x(3x^2 - 5)^{50} dx = \int \frac{1}{6} u^{50} du = \frac{u^{51}}{51 \cdot 6} + C = \frac{(3x^2 - 5)^{51}}{306} + C.$

9 $\int (2x + 3)^4 dx, u = 2x + 3, du = 2 dx, dx = \frac{1}{2} du, \int (2x + 3)^4 dx = \int \frac{1}{2} u^4 du = \frac{u^5}{10} + C = \frac{(2x+3)^5}{10} + C.$

10 $\int \sqrt{3t - 5} dt, u = 3t - 5, du = 3 dt, dt = \frac{1}{3} du. \int \sqrt{3t - 5} dt = \int \frac{1}{3} u^{1/2} du = \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C = \frac{2u^{3/2}}{9} + C = \frac{2(3t-5)^{3/2}}{9} + C.$

15 $\int \sin(4 - x) dx, u = 4 - x, du = -1 dx, dx = -du, \int \sin(4 - x) dx = \int -\sin(u) du = \cos(u) + C = \cos(4 - x) + C.$

17 $\int \sqrt{t}(t^{3/2} + 5)^3 dt, u = t^{3/2} + 5, du = \frac{3}{2} \sqrt{t} dt, \sqrt{t} dt = \frac{2}{3} du. \int \sqrt{t}(t^{3/2} + 5)^3 dt = \int \frac{2}{3} u^3 du = \frac{u^4}{6} + C = \frac{(t^{3/2} + 5)^4}{6} + C.$

18

$$\int \frac{(6x - 9) dx}{(x^2 - 3x + 5)^3}$$

$u = x^2 - 3x + 5, du = (2x - 3) dx, (6x - 9) dx = 3 du$

$$\int \frac{(6x - 9) dx}{(x^2 - 3x + 5)^3} = \int \frac{3 du}{u^3} = \frac{3u^4}{4} + C = \frac{3(x^2 - 3x + 5)^4}{4} + C$$

23 $\int x\sqrt{2x^2 + 1} dx, u = 2x^2 + 1, du = 4x dx, x dx = \frac{1}{4} du. \int x\sqrt{2x^2 + 1} dx = \int \frac{1}{4} u^{1/2} du = \frac{u^{3/2}}{6} + C = \frac{(2x^2 + 1)^{3/2}}{6} + C.$

29 $\int \frac{\ln(x)}{x} dx, u = \ln(x), du = \frac{1}{x} dx. \int \frac{\ln(x)}{x} dx = \int u du = \frac{u^2}{2} + C = \frac{(\ln(x))^2}{2} + C.$

32

$$\int \frac{dx}{x^{2/3}(x^{1/3} + 1)}, u = x^{1/3} + 1, du = \frac{1}{3x^{2/3}} dx, \frac{dx}{x^{2/3}} = 3 du$$

$$\int \frac{dx}{x^{2/3}(x^{1/3} + 1)} = \int \frac{3 du}{u} = 3 \ln(u) + C = 3 \ln(x^{1/3} + 1) + C$$

42 $\int_0^1 x^2(x^3 + 9)^{1/2} dx$, $u = x^3 + 9$, $du = 3x^2 dx$, $x^2 dx = \frac{1}{3} du$, $\int_0^1 x^2(x^3 + 9)^{1/2} dx = \int_9^{10} \frac{1}{3} u^{1/2} du = \frac{2u^{3/2}}{9} \Big|_9^{10} = \frac{2}{9}(10^{3/2} - 9^{3/2}) \approx 1.02728$.

46 Find the area under $y = t\sqrt{t^2 + 9}$ on the interval $[0, 4]$ $A = \int_0^4 t\sqrt{t^2 + 9} dt$, $u = t^2 + 9$, $du = 2t dt$, $t dt = \frac{1}{2} du$, $A = \int_9^{25} \frac{1}{2} u^{1/2} du = \frac{u^{3/2}}{3} \Big|_9^{25} = \frac{1}{3} (25^{3/2} - 9^{3/2}) = \frac{1}{3}(125 - 27) = \frac{98}{3} \approx 32.67$.

51 $\int_{-\pi}^{\pi} \sin(x) dx$. Let $f(x) = \sin(x)$. Then $f(-x) = \sin(-x) = -\sin(x) = -f(x)$, so f is an odd function. Then since the integral is from $-\pi$ to π , by problem 50, $\int_{-\pi}^{\pi} \sin(x) dx = 0$.

53 $\int_{-3}^3 x\sqrt{x^4 + 1} dx$. Let $f(x) = x\sqrt{x^4 + 1}$. Then $f(-x) = -x\sqrt{(-x)^4 + 1} = -x\sqrt{x^4 + 1} = -f(x)$, so f is an odd function and since the integral is from -3 to 3 , problem 50 says that $\int_{-3}^3 x\sqrt{x^4 + 1} dx = 0$.