SPHERE PACKINGS, LATTICES AND GROUP ACTIONS, FALL 2003

Lecture 1
History of the sphere packing problem, lattices in n-dimensional Euclidean space, generalities on lattices.

Lecture 2
Invariants of lattices, generators, volumes, lattices in R^2.

Lecture 3
Lattices, group actions and fundamental domains.

Lecture 4
Equivalence of lattices, sub-lattices, density, kissing numbers.

Lecture 5
Construction and properties of Z^n, n>0, A_n, n>0, D_n, n>2, E_6, E_7, E_8.

Lecture 6
Binary codes, the Golay code and construction of the Leech lattice.

Lecture 7
Uniqueness properties of the Leech lattice.

Lecture 8
The Leech lattice and sphere packings, Moonshine and automorphic forms.

Lecture 9
Root lattices from Lie algebras, proof of uniqueness of E_8: outline of new proof by R. Griess.

Lecture 10
Correspondence between lattices and quadratic forms, classification, automorphism groups of lattices.

Lecture 11
Lorentzian lattices

Lecture 12 (Inna Korchagina)
Sporadic simple groups from the Golay code and the Leech lattice:
M(11), M(12), M(22), M(23), M(24), Co(1), Co(2), Co(3).

Lecture 13
Optimality and uniqueness of lattice packings. Overview of the solutions of Thue, Toth, Segre and Mahler (dim 2), Hales (dim 3), Cohn and Kumar (dim 24).