Sphere packings, lattices and group actions, Fall, 2003
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Hill 425, Wednesdays 9.50am-12.50pm

The problem of packing spheres as densely as possible in Euclidean space has a 400 year history, having been initiated by Johannes Kepler in 1611. Though the problem is still unsolved in general today, attempts to find its solution have led to the discovery of a wealth of mathematics. This course will survey the current status of the sphere packing problem. The following topics will be covered:

Sphere packings and lattices in n-dimensional Euclidean space: overview, history, current
       developments
The related kissing number and covering problems
Lattice packings in 2 and 3 dimensions, solutions to the sphere packing problem
Reflection groups and root lattices, automorphism groups of lattices
The E_8 and Leech lattices
Theta series and modular forms for lattices
Algebraic constructions of lattices
Classification of quadratic forms
Connections with Kac-Moody Lie algebras and vertex operator algebras
Connections with finite simple groups

The main text for the course will be **J.Conway and N.Sloane**, *Sphere Packings, Lattices and Groups*, Springer 1992. This text is now out-of-date relative to many significant recent developments. To supplement this text we will refer to the following:

R.Borcherds, *Automorphic forms and Lie algebras*, in "Current developments in mathematics, Harvard University 1996", Ed. R. Bott, A. Jaffe, S.T. Yau, International Press, 1996.
H. Cohn and N. Elkies *New upper bounds on sphere packings I* Annals of Mathematics 157 (2003), 689-714.
T. Hales, *Cannonballs and honeycombs*, Notices of the AMS 47 (2000), pp 440--449.
J.Milnor and D.Husemoller, *Symmetric Bilinear Forms*, Springer-Verlag, 1973
B. Segre and K. Mahler. *On the densest packing of circles*. Amer. Math. Monthly 51, (1944). 261--270.

Correspondence with Henry Cohn, Noam Elkies and Steve Miller has also brought recent unpublished works to our attention.