TREES AND GROUP ACTIONS

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Fall, 1998

A *tree* X is a non-empty connected graph without circuits. If a group Γ acts on a tree X, certain machinery developed by Bass and Serre permits us to reconstruct the group Γ, the tree X, and the action $\Gamma \times X \longrightarrow X$, giving precise structure theorems for the group Γ.

Conversely, the Bass-Serre theory for reconstructing group actions on trees suggests a technique for *constructing* group actions with prescribed properties, such as *discreteness*, or *minimality*. We describe a method for constructing *tree lattices*; that is, discrete subgroups of finite covolume in the automorphism group of a locally finite tree X, a group which is naturally locally compact, and we discuss the existence theorems for tree lattices.

Of particular interest is the case where the tree $X = X_{r+1,r'+1}$ is homogeneous, of two possible degrees $r+1$ and $r'+1$, where $r = p^s$ and $r' = p^{s'}$, and p is prime. For certain values of s and s', X is the Bruhat-Tits building of a *k-rank 1 group H over a non-archimedean local field k*. The group H has a natural action on its Bruhat-Tits tree X, and we thus obtain a description of the structure of the rank 1 groups of *p-adic type*, and their subgroups, in particular, their lattices, which naturally form deformation spaces.

There are essentially no prerequisites for the course, the necessary background to discuss the above notions will be developed.

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