Practice Test I, Math 291 Spring 2010

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1: Let $\mathbf{x} = (-5, 2, -5)$ and $\mathbf{y} = (1, 2, 1)$.

(a) Is the angle between $\mathbf{x}$ and $\mathbf{y}$ acute or obtuse? Justify your answer.

(b) Find a unit vector $\mathbf{u} \in \mathbb{R}^3$ so that the corresponding Householder reflection $h_{\mathbf{u}}$ reflects $\mathbf{x}$ onto a multiple of $\mathbf{y}$

2: Let $\mathbf{a} = (-3, 2, 2)$ and $\mathbf{b} = (-2, 1, 3)$.

(a) Find an equation for the plane through $\mathbf{0}$, $\mathbf{a}$ and $\mathbf{b}$.

(b) Find the point at which the line through $(-5, 2, -5)$ and $(-4, 3, -4)$ intersects the plane from the first part of the problem.

(c) Find the distance for $\mathbf{0}$ to the line in the second part of the problem.

3: Let $\mathbf{x}(t)$ be the curve given by

$$\mathbf{x}(t) = (2t, t^2, t^3/3) \ .$$

(a) Compute the arc length $s(t)$ as a function of t, measured from the starting point $\mathbf{x}(0)$.

(b) Compute curvature $\kappa(t)$ and torsion $\tau(t)$ as a function of t.

(c) Find an equations for the osculating planes at time $t = 0$ and $t = 1$, and find a parameterization of the line formed by the intersection of these planes.

4: Let $f(x, y)$ be given by

$$f(x,y) = \begin{cases} \dfrac{x^2 \sin(xy)}{x^6 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) \neq (0,0) \ . \end{cases}$$

(a) For any $a, b \in \mathbb{R}$, define the sequence $\{\mathbf{x}_n\}$ by $\mathbf{x}_n = (a/n, b/n)$. Compute $\lim_{n\to\infty} f(\mathbf{x}_n)$.

(b) For any $a, b \in \mathbb{R}$, define the sequence $\{\mathbf{x}_n\}$ by $\mathbf{x}_n = (a/n, b/n^3)$. Compute $\lim_{n\to\infty} f(\mathbf{x}_n)$.

(c) Is the function f continuous? justify your answer.

5: Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x, y) = x^3 y + 2y - 3y^2 x \ .$$

(a) Compute the gradient of f, and find all points (x, y) at which the tangent plane to the graph of f is horizontal.

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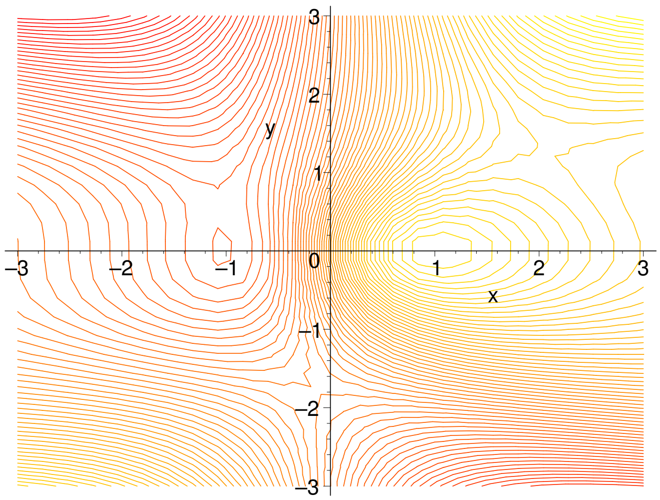
(b) Regard $f(x, y)$ as giving the altitude in a landscape at the point (x, y), and let the positive y-axis point North, and the positive x-axis point East. If you stood at the point $(1, 1)$, and spilled a glass of water, in which compass direction would the water run? Explain your answer.

(c) Find the equation of the tangent line to the level curve of f passing through the point $(1, 1)$.

(d) Could the following be a contour plot of f? Explain your answer.



Extra Credit: Let $\mathbf{x}_0 = (1, 2)$, $\mathbf{v}_1 = (1, 1)$ and $\mathbf{v}_2 = (2, 1)$. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be differentiable, and suppose that

$$f(\mathbf{x}_0 + t\mathbf{v}_1) = 1 + t - 3t^2$$

and

$$f(\mathbf{x}_0 + t\mathbf{v}_2) = 1 - 2t + t^2 - t^3\ .$$

Compute the gradient of f at $\mathbf{x}_0$.