

# Practice Test IB, Math 291 Spring 2010

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**1:** Let  $\mathbf{a} = (1, 1, 1)$

(a) Find a vector  $\mathbf{x}$  such that

$$\mathbf{x} \times \mathbf{a} = (-7, 2, 5) \quad \text{and} \quad \mathbf{x} \cdot \mathbf{a} = 0 .$$

(b) There is no vector  $\mathbf{x}$  such that

$$\mathbf{x} \times \mathbf{a} = (1, 0, 0) \quad \text{and} \quad \mathbf{x} \cdot \mathbf{a} = 0 .$$

Show that no such vector exists.

**2:** Let  $P_1$  denote the plane through the three points  $\mathbf{a}_1 = (1, 2, 1)$ ,  $\mathbf{a}_2 = (-1, 2, -3)$  and  $\mathbf{a}_3 = (2, -3, -2)$ . Let  $P_2$  denote the plane through the three points  $\mathbf{b}_1 = (1, 1, 0)$ ,  $\mathbf{b}_2 = (1, 0, 1)$  and  $\mathbf{b}_3 = (0, 1, 1)$ .

(a) Find equations for the planes  $P_1$  and  $P_2$ .

(b) Parameterize the line given by  $P_1 \cap P_2$ , and find the distance between this line and the point  $\mathbf{a}_1$ .

(c) Consider the line through  $\mathbf{b}_1$  and  $\mathbf{b}_2$ . Determine the point of intersection of this line with the plane  $P_1$ , and find a parametrization of the line given by reflecting this line off plane  $P_1$ .

**3:** Let  $\mathbf{x}(t)$  be the curve given by

$$\mathbf{x}(t) = (e^t \cos(t), e^t \sin(t), e^t) .$$

(a) Compute the arc length  $s(t)$  as a function of  $t$ , measured from the starting point  $\mathbf{x}(0)$ , and find an arc-length parameterization of this curve

(b) Compute curvature  $\kappa(t)$  and torsion  $\tau(t)$  as a function of  $t$ .

(c) Find an equation for the osculating plane at time  $t = 0$

**4:** (a) Let  $f(x, y)$  be given by

$$f(x, y) = \begin{cases} \frac{x}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) . \end{cases}$$

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Does

$$\lim_{(x,y) \rightarrow (0,0)} f(x,y)$$

exist? If so, evaluate the limit. If not, explain why not.

(b) Let  $g(x,y)$  be given by

$$g(x,y) = \begin{cases} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases} .$$

Does

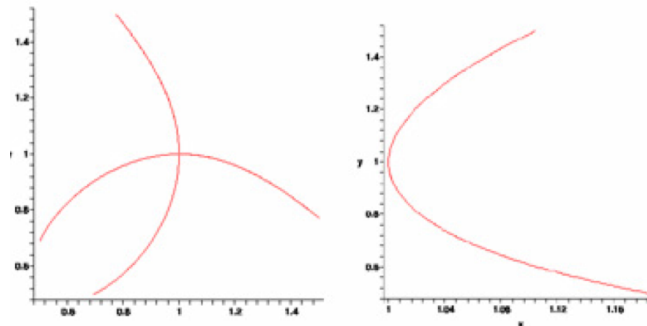
$$\lim_{(x,y) \rightarrow (0,0)} g(x,y)$$

exist? If so, evaluate the limit. If not, explain why not.

5: Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  be given by

$$f(x,y) = x^2y + yx - xy^2 .$$

- (a) Compute the gradient of  $f$ , and find all critical points of  $f$ .
- (b) Find a parameterization of the tangent line to the level curve of  $f$  through the point  $(1,1)$ .
- (c) Find the equation of the tangent plane to the graph  $f$  at the point  $(1,1)$ .
- (d) Could the following be a contour plot of  $f$ ? Explain your answer.



**Extra Credit:** Let  $f(x,y) = 3xy - x^3 - y^3$ . Find all points  $(x,y)$  at which the tangent plane to the graph of  $f$  is orthogonal to the line parameterized by  $t(3,3,1)$ .