Practice Test IIA, Math 291 Spring 2010

Eric A. Carlen[1]
Rutgers University

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1: Let $f(x,y) = x^2 + y^2 - 2yx^2$.

(a) Find all of the critical points of f. Evaluate the Hessian matrix of f at each of these critical points, and determine where each is a local maximum, a local minimum, a saddle, or undecidable from the Hessian.

(b) Sketch a contour plot of f in the vicinity of each of the critical points. Show the computations that lead to the plots to get credit.

2: Let $f(x,y) = (x+y)^4 + (x-y)^2$. Find the minimum and maximum values of f on the unit circle $x^2 + y^2 = 1$, and all of the places on the circle at which f takes on these values.

3: Let Ω be the region in $\mathbb{R}^3$ that is bounded above by the sphere $x^2+y^2+z^2 = 4$, and below by the cone $4z = 4 - \sqrt{x^2+y^2}$. Let $f(x,y,z) = 1/(x^2+y^2+z^2)^2$. Compute $\int_\Omega f(x,y,z)\mathrm{d}V$. To get full credit, you must carry the computations through to the point that only one single variable integral remains to be done. To arrive at this point you will have to choose coordinates appropriately. Making a good choice of coordinates is an important first step; say why you make the choice you make.

4: Let Ω be the region in $\mathbb{R}^2$ that is bounded by the lines

$$y - x = 0 \qquad y - x = 3 \qquad x + 2y = 2 \quad \text{and} \quad x + 2y = 4\ .$$

Compute

$$\int_\Omega \frac{x-y}{x^2+4xy+4y^2}\mathrm{d}A\ .$$

5: Let $\mathcal{S}$ be the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the plane $x + z = 1$.

(a) Compute $\int_\mathcal{S} f(x,y,z)\mathrm{d}S$ where $f(x,y,z) = y/\sqrt{x^2+y^2}$. To get full credit, carry the computations through to the point that only an integral over a single variable remains to be evaluated.

(b) Let $\mathbf{F}$ be the vector field $\mathbf{F}(x,y,z) = (xy, yz, zx)$. Compute the flux integral

$$\int_\mathcal{S} \mathbf{F}\cdot\mathbf{N}\mathrm{d}S$$

where $\mathbf{N}$ is the downward unit normal to the surface. That is, compute the flux across the surface from top to bottom.

Extra Credit: $\mathcal{S}$ be upper hemisphere of the unit sphere in $\mathbb{R}^3$. Let $f(x, y, z) = xyz$. Find the minimum and maximum values of f on $\mathcal{S}$, and all of the points at which f takes on these values. Explain how you are taking into account both of the constraints $x^2 + y^2 + z^2 = 1$ and $z \geq 0$.