

Practice Test IIB, Math 291 Spring 2010

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1: Let $f(x, y) = 3x^3 + 5xy + 5x^2 - 5y^2$.

(a) Find all of the critical points of f . Evaluate the Hessian matrix of f at each of these critical points, and determine where each is a local maximum, a local minimum, a saddle, or undecidable from the Hessian.

(b) Sketch a contour plot of f in the vicinity of each of the critical points. Show the computations that lead to the plots to get credit.

2: Let $f(x, y) = xy$. Find the minimum and maximum values of f the set $D \subset \mathbb{R}^2$ given by

$$(x^2 + y^2)^2 \leq x^2 - y^2 .$$

3: Let Ω be the region in upper right quadrant of \mathbb{R}^2 that is inside the circle $x^2 + y^2 = 1$, and between the parabolas $y = 2x^2$ and $y = 3x^2$. Compute $\int_{\Omega} xy dA$.

4: Let Ω be the region in \mathbb{R}^3 that is the intersection of the three cylinders of unit radius along the three coordinate axes. That is, Ω is the set of points (x, y, z) satisfying

$$y^2 + z^2 \leq 1 \quad x^2 + z^2 \leq 1 \quad \text{and} \quad x^2 + y^2 \leq 1 .$$

Compute the volume of Ω .

5: Let \mathcal{S} be the part of the ellipsoid $4x^2 + 9y^2 + z^2 = 36$ that lies above the plane $z = 3$. Let \mathbf{F} be the vector field $\mathbf{F}(x, y, z) = (x, 0, z)$. Compute the flux integral

$$\int_{\mathcal{S}} \mathbf{F} \cdot \mathbf{N} dS$$

where \mathbf{N} is the downward unit normal to the surface. That is, compute the flux across the surface from top to bottom.

Extra Credit: Given numbers $a > b > c > 0$, find the minimum and maximum values of

$$f(x, y, z) = e^{ax^2 + by^2 + cz^2}$$

on the set

$$x + y + z = 1, \quad x \geq 0, \quad y \geq 0, \quad z \geq 0 .$$

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