Homework Assignment 3 for Math 501, Fall 2010

1. Let \( \mu \) denote Lebesgue measure on \([0,1]\), and let \( \rho \) be a non-negative integrable (with respect to \( \mu \)) function on \([0,1]\). Let \( \nu \) denote the measure given by \( \nu(A) = \int_A \rho(t) d\mu(t) \) for all Borel sets \( A \subset [0,1] \). Define a sequence of real valued functions \( \{p_n\}_{n \geq 0} \) on \([0,1]\) by applying the Gram-Schmidt procedure to the sequence on monomials \( 1, t, t^2, t^3, t^4, \ldots \). Show that for each \( n \), \( p_n \) is a polynomial in \( t \) of degree exactly \( n \), and that \( \{p_n\}_{n \geq 0} \) is an orthonormal basis for \( L^2([0,1], \mathcal{B}, \nu) \).

2. Exercise 43 from Chapter 2 of Folland.

3. Exercise 46 from Chapter 2 of Folland.

4. Exercise 48 from Chapter 2 of Folland.

5. Exercise 50 from Chapter 2 of Folland.