

Multivariable Calculus

Practice Exam

Saturday, June 22, 2002

Chris Long

1. Find all partial derivatives for

(a) $f(x, y) = \frac{2xy}{x^2+y}$

(b) $f(x, y, z) = \ln(x^2 + yz^3)$

2. Given $w = \frac{yx}{\sin(z)}$, $x = u + v$, $y = u - v$, $z = v^2$ find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$.

3. Given $z = \arctan(xy)$, $x = t^2$, $y = t^3$ find $\frac{dz}{dt}$.

4. If $f(x, y) = 3x^2 - 2y^2$, find the directional derivative at $(1, -3)$ from $(1, -3)$ toward $(2, -2)$.

5. Find the maximum value of $f(x, y, z) = xyz$ if $6x + 4y + 3z = 24$.

6. Find the minimum value of $f(x, y) = x^2 - 4xy + 2y^2$ if $x^2 + 2y^2 \leq 1$.

7. Let $z = f(x, y) = x^2y^3 - 2xy$. Find the total differential dz and the tangent plane to $f(x, y)$ at the point $(1, 3)$, and the approximate value of $f(1.1, 3.2)$ using either of these.

8. Determine the local extrema of $f(x, y) = xy^2 - x^2y + x - y$.