

# Multivariable Calculus

## Practice Exam 3

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1. If  $u = e^{x/y}$ ,  $x = 2r - s$ , and  $y = r + 2s$ , find  $\frac{\partial u}{\partial r}$  and  $\frac{\partial u}{\partial s}$ .
2. Find the directional derivative of  $f(x, y) = 4x^2y^4 - 2x + 5$  at the point  $(2, 1)$  in the direction  $\langle -3, 4 \rangle$ .

3. Find

$$\int_0^2 \int_0^1 x^3 y e^{x^2 y^2} dx dy.$$

4. Find the area by the polar curve  $r = 3 + 2 \sin(\Theta)$  for  $0 \leq \Theta \leq 2\pi$ .

5. Find

$$\int_0^1 \int_{2x}^2 e^{y^2} dy dx.$$

6. Find

$$\iint y dx dy$$

over the triangular region with vertices  $(-2, 1)$ ,  $(1, 1)$ ,  $(1, 4)$ .

7. Does

$$\sum_{n=1}^{\infty} \frac{\ln(n^{-3})}{n^{-3}}$$

converge or diverge?

8. Does

$$\sum_{n=1}^{\infty} \frac{\ln(3)}{3n}$$

converge or diverge?

9. Does

$$\sum_{n=1}^{\infty} \frac{n}{3^n}$$

converge or diverge?

10. Find the Taylor series for  $f(x) = e^x$  about the point  $x = 1$ .

11. Find

$$5 \sum_{n=1}^{\infty} \frac{2^{n+2}}{7^{n-2}}.$$

12. Does

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n} + 1}{n + 1}$$

converge or diverge?

13. Does

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n\sqrt{n}}$$

converge or diverge?

14. Does

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^n}{2^{n+2}}$$

converge or diverge?

15. Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} (2x - 4)^n.$$

16. Find

$$\sum_{n=0}^{\infty} \frac{(-x)^{n+1}}{(n+1)!}.$$

17. Find the length of the curve  $x = \cos(e^t)$ ,  $y = \sin(e^t)$ ,  $z = e^t$  for  $0 \leq t \leq 2$ .

18. Find the plane tangent to the surface  $xyz^2 = 1$  at the point  $(1, 1, -1)$ .

19. Given the line  $x+y = x+z = 1$  find the equation of a plane perpendicular to it that contains the point  $(1, 2, 3)$ .
20. Find the angle between the vectors  $\langle 0, -6, 8 \rangle$  and  $\langle 1, 1, 1 \rangle$ .
21. Find the magnitude of the projection of the vector  $\langle 1, 2, 3 \rangle$  onto the vector  $\langle 4, 3, 2 \rangle$ .