Math 251 Multivariable Calculus Fall 2008 Workshop 8 – Exam 2 Review

This list is meant as a guide to reviewing. Please note that any kind of problem included in the syllabus, the list of homework problems, or the workshops can be included in the midterm. Also, that this review sheet is longer than the exam itself.

1. For the given function f, find the critical points and classify each as a relative maximum, a relative minimum, or a saddle point.

$$f(x, y) = x^3 + y^3 + 3x^2 - 18y^2 + 81y + 5$$

2. Find all the critical points of $f(x, y) = x^2y^2$ and show that the Second Derivatives Test fails to classify any of them. Use some other method to determine what each critical point represents.

3. Let $f(x, y, z) = x + y - 3z$ and $g(x, y, z) = 2x^2 + 2y^2 + z^2$. Let S be the surface $g(x, y, z) = 10$.

   (a) Find all points $P(x, y, z)$ on S at which $\nabla f(x, y, z)$ is perpendicular to the tangent plane to S at $P(x, y, z)$.

   (b) Use the result of (a) and the method of Lagrange multipliers to find the maximum and minimum values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = 10$.

4. Evaluate $\displaystyle\int_0^8 \int_{y/4}^2 e^{x^2}\, dx\, dy$.

5. Find the volume under the surface $z = x^2 + y^2$ above the square region bounded by $|x| < 1$ and $|y| < 1$

6. Use a double integral to compute the area of the region which lies inside the rose $r = 4\sin(3\theta)$.

7. Evaluate the triple integral

$$\iiint_{\mathcal{S}} e^z\, dV$$

   where $\mathcal{S}$ is the region described by the inequalities $0 \leq x \leq 1$, $0 \leq y \leq x$, and $0 \leq z \leq x + y$.

8. Use cylindrical coordinates to compute the integral

$$\iiint_R (x^4 + 2x^2y^2 + y^4) dx\, dy\, dz$$

   where R is the cylindrical solid

$$x^2 + y^2 < a^2 \quad \text{with} \quad 0 < z < \frac{1}{\pi}$$

9. Evaluate

$$\iiint_S \frac{dx\,dy\,dz}{\sqrt{x^2+y^2+z^2}}$$

where S is the solid sphere centered at O with radius $\sqrt{3}$.

10. Consider the triple integral

$$\iiint_E f(x,y,z)\,dV = \int_0^{\sqrt{2}/2} \int_{y=x}^{\sqrt{1-x^2}} \int_{z=-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} f(x,y,z)\,dz\,dy\,dx.$$

   (a) Give a set of inequalities on x, y, and z that define the solid E. Describe the *boundary surfaces* of E. Sketch the solid E and the region R in the first quadrant of the xy-plane that E projects onto. Then describe E by a set of inequalities in *spherical coordinates* (ρ, ϕ, θ).

   (b) Suppose $f(x,y,z) = x^2+y^2+z^2$. Change the triple integral to spherical coordinates and evaluate it (remember that the element of volume is $dV = \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$).

11. Evaluate the integral of the function $f(x,y,z) = \sin(\sqrt{x^2+y^2+z^2}^3)$ over the region in the first octant which is outside the cone $z = \sqrt{x^2+y^2}$ and inside the sphere $x^2+y^2+z^2=2$.