Math 251 Multivariable Calculus Fall 2008 Workshop 10

You will write up *one* of these problems and turn it in at next week's workshop. You will be graded on your exposition as well as on the mathematical content of your report. During the workshop period, you are encouraged to work with other students in your group. However, the writeup you turn in should represent your own presentation of the solution. It should not be a (rewritten) version of someone else's work. You should:

- Work out the problem and have a clear plan of presentation before you begin to write up your final solution.
- Be neat and write legibly. Don't try to squeeze your report onto this piece of paper.
- Show all steps. Explain what you are doing at each step in complete sentences.

Write as if you intended your explanation to be part of a good textbook, or of a professional lab report in science or engineering.

1. Let C be the curve from $(1,0)$ to $(4,3)$ that follows the arc of the circle $x^2 + y^2 = 1$ from $(1,0)$ to $(0,1)$ and then the line segment from $(0,1)$ to $(4,3)$.

   (a) Graph C and find parametric formulas for C (split C into two parts). Then express dx and dy in terms of your parameters. (you will need to give the formulas piecewise).

   (b) Use the result of part (a) to write the line integral

   $$\int_C x\sqrt{y}\,dx + 2y\sqrt{x}\,dy,$$

   as a sum of two definite integrals (with the parameter as the variable of integration). Then evaluate these integrals.

2. Let C be the portion of the parabola $y = x^2$ from $(-1,1)$ to $(2,4)$.

   (a) Sketch C and express the differential of arclength ds in terms of x and dx on C.

   (b) Use part (a) to write line integral $\int_C x\,ds$ as a definite integral, and evaluate it.

   (c) Let $\mathbf{F}(x,y) = x\sin y\,\mathbf{i} + y\,\mathbf{j}$. Express the work integral $\int_C \mathbf{F}\cdot d\mathbf{s}$ as a definite integral and evaluate it.

   (d) Let $\mathbf{F}(x,y) = \nabla f(x,y)$, where $f(x,y) = e^{xy}$. Evaluate the work integral $\int_C F\cdot d\mathbf{s}$. (*Hint:* There is an easy way.)

3. Suppose that $\mathbf{F}(x,y)$ is a *constant* vector field: $\mathbf{F}(x,y) = \mathbf{v}$ for all (x,y) and some fixed nonzero vector $\mathbf{v}$. Let C be a straight line path described parametrically by $\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{u}_0 t$ for $0 \le t \le 1$

   (a) Sketch this situation and calculate the work integral $\int_C \mathbf{F}\cdot d\mathbf{s}$.

(b) When is the work integral in (a) negative? When is it zero?

4. Let $\mathbf{F} = ye^{xy}\mathbf{i} + (xe^{xy} - 2y)\mathbf{j}$

(a) Show that $\mathbf{F}$ is a conservative vector field.

(b) Find a potential function for $\mathbf{F}$.

(c) Suppose C is the curve given parametrically by

$$\mathbf{r}(t) = \left(\ln(\sin(t^4) + 1)(\cos(e^{\cosh(t)}))\right)\mathbf{i} + \left(\cos(t^4)e^{\sin(t^4)e^{24t^6}}\right)\mathbf{j}$$

for t from 0 to $\sqrt[4]{\pi}$. Find

$$\int_C ye^{xy}\,dx + (xe^{xy} - 2y)\,dy$$

5. Suppose that $\mathbf{F}$ is the force field defined by

$$\mathbf{F}(\mathbf{r}) = \frac{c\,\mathbf{r}}{|\mathbf{r}|^3},$$

where c is a constant and $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$. (Note that $\mathbf{r}$ is a vector field; it is not the polar coordinate variable.)

(a) Show that $\mathbf{F} = \nabla f$, for some function $f(x, y, z)$, and find such a function f.

(b) Show that the work done by $\mathbf{F}$ in moving an object from a point P_1 along a path to a point P_2 is given by

$$c\left(\frac{1}{d_1} - \frac{1}{d_2}\right),$$

where d_1 and d_2 are the respective distances of the points P_1 and P_2 from the origin.