

The exam material:

The exam will cover Chapters 3 and 4 and Section 5.1 from the text with *less* emphasis on the following sections: the subsection of 3.3 on “An Application,” Section 4.5, and the material from Section 5.1 on diagonalization. (The rest of the material from 5.1 on eigenvalues and eigenvectors *will* be covered).

*The exam will consist almost entirely of problems taken directly from the lists of practice problems below. You will not be asked to directly prove a difficult theorem this time, but you should know the statements of the theorems listed below.*

Practice Problems from the Book:

3.2#6a, 3.2#21, 3.3#9, 3.4#4, 3.4#5, 3.4#9, 3.4#10, 3.4#15, 4.2#3, 4.2#19, 4.2#25, 4.2#30, 4.3#10, 4.3#11, 4.3#12, 4.3#15, 4.3#17, 5.1#3c, 5.1#6, 5.1#7, 5.1#8, 5.1#12, 5.1#14, 5.1#15.

Other Practice Problems:

1. In each of the following situations, either give an example or explain why none could exist. A system of  $m$  distinct linear equations and  $n$  unknowns with
  - i. exactly one solution with  $m < n$ .
  - ii. exactly one solution with  $m > n$ .
  - iii. no solutions with  $m < n$ .
  - iv. infinitely many solutions with  $m > n$ .
  - v. exactly two solutions.
2. Let  $A = \begin{bmatrix} 1 & 2 & 8 \\ 4 & 5 & 23 \end{bmatrix}$  and  $R = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}$ . Find a matrix  $B$  such that  $BA = R$  or explain why none could exist.
3. Suppose  $A$  is an  $n \times n$  real nilpotent matrix (i. e.  $A^k = O$  for some integer  $k > 0$ ).
  - (a) Compute the characteristic polynomial for  $A$ . (*Hint:* Use eigenvalues.)
  - (b) Prove that  $A + I$  is invertible.
  - (c) Generalize the result in part (b).

Important Theorems to Know:

3.1, Cor. 2 on pg. 158, 3.16, all the results in Section 4.4, 5.2, 5.4.

Definitions to Know:

Rank of a matrix or a linear transformation. Consistent or inconsistent systems of equations. Homogeneous systems and inhomogeneous systems. Determinant of a matrix and a linear transformation. Eigenvalue, eigenvector and eigenspace. Characteristic polynomial.

Studying for the exam:

Do all the problems above! It might seem like a lot of problems but you have already done many of them on the homework. If you get stuck then work with some of your classmates or come to the review session or office hours. I will happily go over any of these problems or any other questions people have.