1. Which of the following functions have an *isolated* singularity at $z = 0$? Explain carefully.

(a) $f(z) = \sqrt{z}$.

(b) $f(z) = |z|$

(c) $f(z) = \exp(-1/z^2)$

2. The Laurent expansion of a function $f(z)$ is given by $f(z) = \sum_{k=-\infty}^{\infty} \frac{z^k}{|k|!}$.

(This series converges in $\mathbb{C} \setminus \{0\}$.)

(a) What type of singularity does f have at $z = 0$? Explain carefully.

(b) Find $\oint_C f(s)ds$ where C is the circle of radius 3 centered at the origin.

3. Find the first two nonzero terms of the Laurent expansion of the function $f(z) = \dfrac{1}{(\sin(z) + z^3/6)^2}$ at $z = 0$.

4. Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{1}{(x^2+1)(x^2+4)} dx$$

Justify the steps in your calculation.

5. In the following three questions you are asked to explain carefully whether the statements are true or false:

(a) There exists a function $f(z)$ which is analytic in $\mathbb{C} \setminus \{z_1, z_2\}$, real on $\mathbb{R}^+$ and which has two poles in the lower half plane, at z_1 and z_2.

(b) There exists a function $f(z)$ which is analytic in the upper half plane, and whose limit as $z = x + iy$ approaches the real line is $\lim_{y \to 0^+} f(x + iy) = |x|$.

(c) There exists a function $f(z)$ which is analytic in $\mathbb{C} \setminus \{1\}$, real-valued on $\mathbb{R} \setminus \{1\}$ and which has a simple pole at $z = 1$.

6. Are there any values of z for which $|\sin(\sin(z))| > 1$? Explain.

7. Show that the function $g(x, y) = x^2 + x - y^2$ is harmonic in $\mathbb{C}$ and find a harmonic conjugate of g.

8. Find a conformal mapping of the right half plane $z : \Re(z) > 0$ onto the disk $\{z = x + iy : (x-1)^2 + y^2 < 4\}$.