

1. Let $f(z) = \frac{e^{z^2} - 1}{z^3}$ defined for all complex numbers z except $z = 0$.
- (a) Is $z = 0$ an isolated singular point of f ? If so, is it a removable singularity, a pole, or an essential singularity of f ?
- (b) Let g be analytic in the unit disk and such that $g(0) = 1$ and $g(1/2) = 1/2$. Show that there must exist a point z in the unit disk such that $|g(z)| > 1$.

2. Evaluate the integrals

(a)
$$\int_0^{2\pi} \frac{d\theta}{(4 - \cos(\theta))^2}$$

(b)
$$\int_{-\infty}^{\infty} \frac{e^{ix}}{x^2 + x + 1} dx$$

(c)
$$\int_{-\infty}^{\infty} \frac{\sin(x)}{x^2 + x + 1} dx$$

(d)
$$\int_0^{\infty} \frac{x^{-1/3}}{x + 1} dx$$

3. (a) Find the number of zeros of the polynomial $f(z) = z^3 + z^2 + 2$ inside the unit disk $|z| \leq 1$.
- (b) Find the number of zeros of the polynomial in part (a) in the annulus $1 \leq |z| \leq 2$.

4. Let $f(z) = (z^2 + 2)/(z^2 + 4)$. Find the maximum value of $|f(z)|$ inside the unit disk $|z| \leq 1$. Find the points for which this maximum value is attained.