Math 250-C1 Spring 2003 Practice Exam 1 (Woodward)

PROBLEM 1 Find (a) a row echelon form and (b) the reduced row echelon form and (c) an LU factorization for the matrix

$$\begin{bmatrix} 1 & -1 & 0 & -8 & -3 \\ -2 & 1 & 0 & 9 & 5 \\ 3 & -3 & 0 & -2 & -11 \end{bmatrix}$$

PROBLEM 1' Using elimination find (a) a row echelon form (b) the reduced row echelon form

$$A = \begin{bmatrix} 1 & 3 & -1 & 1 & 0 \\ 2 & 6 & -1 & 4 & 15 \\ -1 & -3 & 2 & -1 & 15 \end{bmatrix}.$$

(c) Write the elementary matrices corresponding to the row-operations you performed in (a). (d) Find an LU-factorization for A.

(e) Using your answer to (b), find all solutions to the system whose augmented matrix is A, that is,

$$\begin{aligned} x_1 + 3x_2 - x_3 + x_4 &= 0 \\ 2x_1 + 6x_2 - x_3 + x_4 &= 15 \\ -x_1 + -3x_2 + 2x_3 - x_4 &= 15 \end{aligned} \quad .$$

PROBLEM 2 True or False? Justify your answer.

(a) If A and B are matrices such that $AB = I$ then A and B are invertible.

(b) If the number of columns of A is more than the number of rows, then $Ax = b$ has infinite solutions.

(c) The nullity of a matrix is the number of free variables.

(d) The inverse of an invertible upper triangular matrix is upper triangular.

(e) If A is invertible then A^T is invertible.

(f) If A is invertible then $ref(A)$ is invertible.

(g) If the number of columns of A is equal to the number of rows, then $Ax = b$ always has a unique solution.

(h) A matrix whose diagonal entries are all zero is not invertible.

(i) The identity matrix I is invertible.

(j) The inverse of a symmetric matrix is symmetric.

(k) If A and B are symmetric, then AB is symmetric.

(l) If the number of columns of A is greater than the number of rows, then $Ax = b$ cannot have a unique solution.

(m) If A and B are symmetric matrices, then $A + B$ is also symmetric.

(n) If $v_1, \ldots, v_m$ are vectors in R^n, and $m \geq n$, then these vectors span R^n.

(o) If A is invertible, then A is a product of elementary matrices.

PROBLEM 3 Compute (a) A^2 (b) A^{-1} and (c) A^T for $A = \begin{bmatrix} 0 & 0 & 3 \\ 0 & 1 & 0 \\ 1/3 & 0 & 0 \end{bmatrix}$

(or $A = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 1 & 2 \\ 3 & 0 & 0 \end{bmatrix}$. or $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$.) You might want to check your answer by multiplying A by A^{-1}.

PROBLEM 4 Find the unique quadratic (degree 2) polynomial passing through the points $(0, 1), (1, 2), (-1, 3)$.

PROBLEM 5 Find a set of vectors that is as small as possible that has the same span as $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

PROBLEM 6 Prove that (a) if the equation $Ax = b$ has more than one solution, then it has infinitely many solutions.

(b) if v_1 and v_2 are linearly independent, then v_1 and $v_1 + v_2$ are linearly independent.

(c) Prove that if v_1 and v_2 span a subset V, then so do v_1 and $v_1 + v_2$.

(d) Prove that if a matrix A is invertible, then A^2 is also invertible.