Practice Exam 1 for Math 250, Fall 2000

Problem 1 Find all solutions to the system

$$x_1 + x_2 - 2x_3 + x_4 = 6$$
$$2x_1 - x_2 + x_3 - 3x_4 = 0.$$

What are the solutions to the corresponding homogeneous system?

Problem 2 (a) Find the distance between the points $(3, 1, 2, 4)$ and $(-1, 2, 1, 2)$ in R^4.

(b) Find a non-zero vector perpendicular to $v = [3\ 1\ 2\ 4]$ in R^4.

(c) Find the angle between the vectors $v = [3\ 1\ 2\ 4]$ and $w = [-1\ 2\ 1\ 2]$.

Problem 3 Find a basis for the row-space, the column-space, and the null-space of the matrix

$$A = \begin{bmatrix} 0 & 6 & 6 & 3 \\ 1 & 2 & 1 & 1 \\ 4 & 1 & -3 & 4 \\ 1 & 3 & 2 & 0 \end{bmatrix}.$$

What are the rank and nullity of A?

Problem 4 True or False? Explain your answer.

(a) The number of columns containing pivots in rref(A) is the same as the number of rows containing pivots.

(b) If A and B are symmetric, then $A + B$ is symmetric.

(c) If the number of columns of A is greater than the number of rows, then $Ax = b$ cannot have a unique solution.

(d) The column space of rref(A) is the same as the column space of A.

Problem 5 Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & 0 & 3 \end{bmatrix}$.

Compute AB.

Problem 6 Find the inverse of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$

(Matrices of this form are called *upper triangular*.) You might want to check your answer by multiplying A by A^{-1}.

(b) Find the transpose of A.

Prove or disprove (i.e. give a counterexample) the following statements. Recall, a matrix is symmetric if $A = A^T$.

(a) If A and B are symmetric matrices, then $A + B$ is also symmetric.

(b) If A and B are symmetric matrices, then AB is also symmetric.

Problem 7 Find the LDU factorization for the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}.$$

Problem 8 Determine which of the following are subspaces, and determine their dimension. Show your work! (a) The set of vectors perpendicular to [2 3 5 8]. (b) The set of invertible 2 by 2 matrices.