Quantum non-abelian localization: Conjectures and partial results

Chris Woodward,Rutgers

joint with E. Gonzalez

Question: how are equivariant Gromov-Witten invariants of a Hamiltonian G-manifold related to those of its symplectic quotient?

thanks to C. Teleman

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i.e. triples (P, A, u) of principal G-bundle P, connection A, and holomorphic $u : \mathbb{P}^1 \to P \times_G X$

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- moment map $\mathcal{A}(P, X) \to \Omega^2(\mathbb{P}^1, P(\mathfrak{g})), (A, u) \mapsto F_A + \mathrm{Vol}\, u^* \Phi$
- Depends on two-form $\mathrm{Vol} \in \Omega^2(\mathbb{P}^1)$ and inner product $\mathfrak{g} \times \mathfrak{g} \to \mathbb{R}$.

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Question: how does $\overline{M}(P,X)$ depend on ϵ?

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In between have wall-crossing formulas, for example Kalkman

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Theorem: $\overline{M}(X)_\infty$ is a compactification of $\overline{M}(P, X)_\epsilon, \epsilon \to \infty$.

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Integrate to get $\int_{\mathbb{P}^1} u_\alpha^* \Phi \, \mathrm{Vol}_{\mathbb{P}^1} \to 0$

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Conj 3: In between, have wall-crossing $\langle \alpha \rangle_{\epsilon_1} - \langle \alpha \rangle_{\epsilon_0} = \sum_{\epsilon \in (\epsilon_0, \epsilon_1), \psi \in \mathfrak{g}} (\# W_\psi / \# W) \langle \alpha \rangle_{X^\psi, X, (\mathfrak{g}/\mathfrak{g}_\psi)^{\mathbb{C}}, TX/TX^\psi, \epsilon}$

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Wall-crossing terms count vortices in X^ψ, twisted by Euler classes of index bundles $(\mathfrak{g}/\mathfrak{g}_\psi)^{\mathbb{C}}$ and TX/TX^ψ, and allowing sphere bubbling in X

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In other words, we believe we have proved Conjectures 1 and 3.

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Quantum non-abelian localization in general would imply genus zero quantum Martin with Kirwan replaced by quantum Kirwan