Math 477, Sec. E3

Name:______________________

Ch. 1 problem, due Wednesday, June 27

According to p.15, if $n_1 + n_2 + \cdots n_r = n$, then $\binom{n}{n_1, n_1, \ldots, n_r}$ represents the number of possible divisions of n distinct objects into r distinct groups of respective sizes n_1, n_2, ..., n_r.

How many ways can we divide 5 objects, a, b, c, d and e, into two groups, with one group of size 2 and the other group of size 3?

$$\text{Answer: } \binom{5}{2, 3} = \frac{5!}{2!3!} = 10.$$

Now, how many ways can we divide 5 objects, a, b, c, d and e, into three groups, with two groups each of size 1 and one group of size 3?

One might reply: the answer is $\binom{5}{1,1,3} = \frac{5!}{1!1!3!} = 20$.

But someone else might reply like this: an example of a "way" of dividing the objects is

$$W = \Big\{\{c\}, \{a, b, e\}, \{d\}\Big\}.$$

We can uniquely identify this "way" by the group of three: $W = W_{\{a,b,e\}}$. Same goes for every other way of dividing the objects: the subgroup of three uniquely determines the way. Therefore,

$$\begin{aligned} \text{the number of ways } &= \text{ the number of combinations of 5 objects taken 3 at a time} \\ &= \binom{5}{3} = \frac{5!}{3!2!} = 10. \end{aligned}$$

Explain this discrepancy.