1. Evaluate \( \lim_{x \to 2^+} \frac{1}{(x-2)^3} \) and \( \lim_{x \to 2^-} \frac{1}{(x-2)^3} \). Use these to evaluate \( \lim_{x \to 2} \frac{1}{(x-2)^3} \).

**Solution:** As \( x \to 2 \) from the right \( x - 2 \) becomes a smaller and smaller *positive* number. So \( (x-2)^3 \) is a smaller and smaller positive number and therefore \( 1/(x-2)^3 \) becomes a large positive number. So \( \lim_{x \to 2^+} \frac{1}{(x-2)^3} = \infty \).

As \( x \to 2 \) from the left \( x - 2 \) becomes a smaller and smaller *negative* number. Therefore \( 1/(x-2)^3 \) becomes a larger negative number. So \( \lim_{x \to 2^-} \frac{1}{(x-2)^3} = -\infty \).

Recall that a limit only exists if the right and left limits match. These limits do not match. So \( \lim_{x \to 2} \frac{1}{(x-2)^3} \) does not exist.

Notice that the power in the denominator, 3, is an odd number. Thus when we cubed \( (x-2) \) when \( x \) approached from the left we were cubing a negative number and it stayed negative. What would happen if the power was even? For example, compute: \( \lim_{x \to 2} \frac{1}{(x-2)^4} \).

2. Evaluate \( \lim_{x \to 4} \frac{x^2 + x - 20}{(x-1)^2 - 9} \).

**Solution:** If we attempt to plug in \( x = 4 \) to the expression we get \( \frac{0}{0} \). This suggests we need to do some algebraic simplification and then try evaluating again. We can factor the numerator and denominator as

\[
\frac{x^2 + x - 20}{(x-1)^2 - 9} = \frac{x^2 + x - 20}{x^2 - 2x - 8} = \frac{(x-4)(x+5)}{(x-4)(x+2)} = \frac{x+5}{x+2}.
\]

Notice it is relatively easy to factor the numerator and denominator since we already know that 4 is a root of each polynomial so we know the factor \( (x-4) \) will appear in both factorizations.

So we find \( \lim_{x \to 4} \frac{x^2 + x - 20}{(x-1)^2 - 9} = \lim_{x \to 4} \frac{x+5}{x+2} = \frac{4+5}{4+2} = \frac{9}{6} = \frac{3}{2} \).