For this quiz $A = \begin{bmatrix} 1 & 1 & 4 \\ 3 & 3 & 12 \\ 2 & 0 & 2 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $b = \begin{bmatrix} t \\ 1 \end{bmatrix}$.

1. Calculate the reduced row echelon form of the matrix given below for the system of linear equations $A\vec{x} = \vec{b}$. Indicate each elementary row operation that you use.

Solution:

$$\begin{bmatrix} 1 & 1 & 4 & 1 \\ 3 & 3 & 12 & t \\ 2 & 0 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 4 & 1 \\ 0 & 0 & 0 & t-3 \\ 0 & 0 & 0 & t-3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & t-3 \end{bmatrix}$$

Now if $t = 3$ then the matrix above is in reduced row echelon form: $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

However, if $t \neq 3$, then we can perform the row operation $\frac{1}{t-3}r_3 \rightarrow r_3$ and obtain

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

which is in rref (and not consistent!).

2. The equation $A\vec{x} = \vec{b}$ is consistent when $t = 3$. For this value of $t$ the general solution has 1 free variables. (Fill in the blanks with the correct numbers.

3. For the value of $t$ in (2) find the general solution $\vec{x}$ to $A\vec{x} = \vec{b}$. Write the solution in vector form.

Solution: $x_3$ is a free variable. The first equation is $x_1 + 2x_3 = 0$. So $x_1 = -2x_3$. Similarly, $x_2 = 1 - 3x_3$. Thus the general solution is

$$\begin{bmatrix} -2x_3 \\ 1 - 3x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix}.$$ 

Either of these two is an acceptable answer.
4. What is $\text{rank}(A)$? What is the nullity of $A$?

**Solution:** Note that in computing the rref of $[A \vec{b}]$ we have computed the rref of $A$:

$$
\begin{bmatrix}
1 & 0 & 2 \\
0 & 1 & 3 \\
0 & 0 & 0
\end{bmatrix}.
$$

Thus the rank of $A$ is 2. So the nullity is $3 - 2 = 1$. Note that we had 2 basic variable in our solution above and 1 free variable.