Math 251, Midterm Two Review Questions

1. Find the max and min values of \( f(x, y, z) = x + y + 2z \) on the ellipsoid \( x^2 + 4y^2 + 9z^2 = 1 \).

2. Consider the "cone frustum" show below

![Figure 1: Don't get FRUSTrated!](image)

Compute the volume of this cone by parameterizing the region and setting up a triple integral (in whatever coordinate system you like). Check your answer by computing the volume with simple geometry.

3. Let \( \mathbf{F}(x, y, z) = \langle ye^{xy} + 2xz, xe^{xy}, 2xz + 3z^2 \rangle \). Is \( \mathbf{F} \) a conservative vector field? Either explain why not or show that it is by finding a function \( f : \mathbb{R}^3 \to \mathbb{R} \) such that \( \mathbf{F} = \nabla f \).

4. Repeat the question above for the vector field

\[
\mathbf{G}(x, y, z) = \langle yz \cos(xy), xz \cos(xy), x^2 \cos(xy) \rangle.
\]

5. Evaluate \( \iiint_E \frac{9}{\pi} (x^2 + y^2)^3 \, dV \) where \( E \) is the solid that lies within the cylinder \( x^2 + y^2 = 1 \), above the \( xy \)-plane and below the cone \( z^2 = 4x^2 + 4y^2 \).

6. Evaluate the iterated integral below by converting to polar coordinates.

\[
\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{-3}^{3} x^2 y^3 \, dy \, dx
\]

7. Let \( T \) be the triangle with vertices \((-10, 0), (10, 0) \) and \((0, 20)\). Note that \( T \) is not equilateral. Let \( S \) be the circle bounded by \((x - 1)^2 + (y - 3)^2 = 4\). Let \( R \) be the region inside \( T \), but outside \( S \) (a triangle with a disk cut out of it). Find the coordinates of the center of mass of \( R \).