1. A particle travels along the curve \( r(t) = (\cos(t), \sin(t), t) \) as \( t \) ranges from 0 to \( 8\pi \). Describe this path in words. How far did the particle travel? Compare this to the particle’s net displacement over this time period.

**Solution:** Notice that \((\cos(t), \sin(t))\) as \( t \) ranges from 0 to \( 8\pi \) does 4 laps around the unit circle. However, as \( t \) increases, the \( z \)-coordinate of \( r(t) \) increases. So the curve is a helix (a slinky toy is a good real world example). There is a picture of a piece of this curve on page 760 of your text book. See figure 11. To compute how far the particle traveled we first compute \( r'(t) = (-\sin(t), \cos(t), 1) \) and so \( \|r'(t)\| = \sqrt{\sin^2(t) + \cos^2(t) + 1} = \sqrt{1 + 1} = \sqrt{2} \). The distance traveled is given by the arc length integral:

\[
\int_0^{8\pi} \|r'(t)\| \, dt = \int_0^{8\pi} \sqrt{2} \, dt = 8\sqrt{2} \pi.
\]

Next, we compute the net displacement of the particle over this time period. At \( t = 0 \), the particle has position \((1, 0, 0)\). At \( t = 8\pi \), the particle has position \((1, 0, 8\pi)\). So the net displacement is \( 8\pi \). So the path of the particle is \( \sqrt{2} \) times as long as its net displacement.

2. Find the curvature of \( r(t) = (t, t^2, t^3) \) at \( t = 2 \).

**Solution:** We use the formula \( \kappa(t) = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3} \). Note that to use this formula, we need \( r'(t) \neq \vec{0} \), i.e. we need a regular parameterization. We compute \( r'(t) = (1, 2t, 3t^2) \) and \( r''(t) = (0, 2, 6t) \). Note \( r'(t) \neq \vec{0} \). We compute \( r'(t) \times r''(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix} = (2, 6t, 6t^2) \). So

\[
\|r'(t) \times r''(t)\| = \sqrt{36t^2 + 36t^2 + 4} \quad \text{and} \quad \|r'(t)\| = \sqrt{9t^4 + 4t^2 + 1}.
\]

So \( \kappa(t) = \frac{\sqrt{36t^4 + 36t^2 + 4}}{(9t^4 + 4t^2 + 1)^{3/2}} \).

We find

\[
\kappa(2) = \frac{\sqrt{724}}{161^{3/2}} = \frac{2\sqrt{181}}{161^{3/2}}.
\]