1. Let $D$ be the parallelogram spanned by vectors $\langle 7, 2 \rangle$ and $\langle 4, 4 \rangle$ anchored at the origin. Let $G(u, v) = \langle 7u + 4v, 2u + 4v \rangle$.

a) Show that $G$ maps the square $[0, 1] \times [0, 1]$ to $D$.

**Solution:** We know linear maps send parallelograms to parallelograms. We just need to check that the spanning vectors of the unit square, $\langle 0, 1 \rangle$ and $\langle 1, 0 \rangle$, get mapped by $G$ to the spanning vectors of $D$. We have $G(1, 0) = (7, 2)$ and $G(0, 1) = (4, 4)$ so this is indeed the case.

b) Use the Change of Variables Formula and $G$ to compute $\int\int_D e^{x+y} dA$.

**Solution:** First we compute the Jacobian: $\begin{vmatrix} 7 & 2 \\ 4 & 4 \end{vmatrix} = 28 - 8 = 20$. So by the Change of Variables Formula we have

$$\int\int_D e^{x+y} dA = \int_0^1 \int_0^1 e^{9u+8v} \cdot 20 \ du \ dv$$

$$= 20 \int_0^1 \frac{1}{9} (e^9 - 1) e^{8v} \ dv$$

$$= \frac{20}{9} (e^9 - 1) \cdot \frac{1}{8} (e^8 - 1)$$

$$= \frac{5}{18} (e^9 - 1)(e^8 - 1)$$