1. Let \( F(x, y, z) = (xyz, x^2, y^2 + z) \).

a) Show that \( H = \nabla \times F = (2y, xy, 2x - xz) \).

**Solution:**

\[
\nabla \times F = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
xyz & x^2 & y^2 + z
\end{vmatrix} = (2y - 0, -(0 - xy), 2x - xz) = (2y, xy, 2x - xz).
\]

b) Compute \( \int \int_S H \cdot dS \) where \( S \) is the hemisphere \( x^2 + y^2 + z^2 = 1, \ z \geq 0 \) oriented with outward pointing normals.

**Solution:** The boundary of this surface is the unit circle in the \( xy \)-plane. We can parameterize this as \( c(t) = (\cos t, \sin t, 0) \) where \( 0 \leq t \leq 2\pi \). Note that this orientation is consistent with outwardly facing normals. Using Stoke’s theorem we have \( \int \int_S \nabla \times F \cdot dS = \oint_C F \cdot dS \).

Now \( c'(t) = (-\sin t, \cos t, 0) \). So we can evaluate this line integral as

\[
\oint_C F \cdot dS = \int_0^{2\pi} (0, \cos^2 t, \sin^2 t) \cdot (-\sin(t), \cos(t), 0) dt = \int_0^{2\pi} \cos^3 t = 0.
\]

2. State the divergence theorem. [Do your best, you don’t have to remember every condition to get full credit, but your answer shouldn’t be just an equation]

**Solution:** Let \( S \) be a closed surface that encloses a region \( W \) in \( \mathbb{R}^3 \). Assume that \( S \) is piecewise smooth and is oriented by normal vectors pointing to the outside of \( W \). Let \( F \) be a vector field whose domain contains \( W \). Then

\[
\int \int \int_W \nabla \cdot F \ dV.
\]

Again, I am not looking for every word of this theorem to appear. You should explain that \( S \) is a surface which contains a region \( W \) and comment on the orientation (since this matters for the surface integral in the formula).