

Oral Qualifying Exam Syllabus

Colleen Duffy

28 February 2006

Committee: Profs. R. Wilson, R. Goodman, V. Retakh, E. Taft

Major Topic: Noncommutative Algebra

1. General noncommutative ring theory ([Lam] §§1-4, 7-9)
 - Semi-simple modules and rings
 - Wedderburn-Artin Theory
 - Jacobson Radical
 - modules over kG /representations of finite groups, characters
 - linear groups: Burnside's Theorem
2. Quasideterminants ([GGRW])
 - definition - in terms of inverses and recursive
 - properties - e.g. row/column relations, Sylvester's theorem
 - applications - e.g. Vandermonde quasideterminant, Vieta theorem, symmetric functions, quasi-Plucker coordinates
3. Algebras related to roots of equations
 - $Q_n, A(\Gamma)$ - definition, describe linear basis[GRSW]
 - Bergmans Diamond Lemma ([Bergman] §§1-3)
 - factorization of twisted polynomial rings: remainder and product theorems, definition and example of Wedderburn polynomials ([LL] §§1-2; [LL2] §§1-3)
 - Koszul algebras: definition, dual of, Hilbert series of ([Froberg] §§1-2)
4. Lie Algebras ([GW] Ch. 1-2) ([Humphreys] §§1-8)
 - Lie group and Lie algebra correspondence
 - classification of finite dimensional semisimple algebras over the complex numbers
 - classification of irreducible representations by highest weight (isomorphism)
 - some explicit examples
 - PBW Theorem

Minor Topic: Hopf Algebras

- Definition of Hopf algebra (coalgebra, bialgebra, antipode)
- Coideals and comodules
- Duality - A^0, C^*
- Definition of integrals and smash product
- H-module algebra and coalgebra
- Fundamental Theorem of Coalgebras

References

- Bergman Bergman, George M. "The Diamond Lemma for Ring Theory." *Advances in Mathematics*, 29 (1978) 178-195.
- Froberg Froberg, R. "Koszul Algebras." 1998.
- GGRW Gelfand, Israel, Sergei Gelfand, Vladimir Retakh, and Robert Wilson. "Quasideterminants." *Advances in Mathematics* 193 (2005) 56-141.
- GRSW Gelfand, Israel, Vladimir Retakh, Shirlei Serconek, and Robert Wilson. "On a Class of Algebras Associated to Directed Graphs."
- GW Goodman, Roe and Nolan Wallach. Representations and Invariants of the Classical Groups. *Encyclopedia of Mathematics and its Applications*, Cambridge, 1998.
- Humphreys Humphreys, James E. Introduction to Lie Algebras and Representation Theory. Springer-Verlag, New York, 1972.
- Lam Lam, T.Y. A First Course in Noncommutative Rings. 2nd ed. Springer-Verlag, New York, 2001.
- LL Lam, T.Y. and A. Leroy. "Vandermonde and Wronskian Matrices over Division Rings." *Journal of Algebra* 119, 308-336 (1988).
- LL2 Lam, T.Y. and Andre Leroy. "Wedderburn Polynomials over Division Rings, I." *Contemporary Mathematics*.