

1. Simplify each of the following by performing the indicated operation(s).

(a) $(a^{1/2} - b^2)(a^{1/2} + b^2)$

There are 2 ways to do this problem:

- You can notice that it is in the form of “difference of squares”:

$$\begin{aligned}(a^{1/2} - b^2)(a^{1/2} + b^2) &= (a^{1/2})^2 - (b^2)^2 \\ &= a^{(1/2) \cdot 2} - b^{2 \cdot 2} \\ &= a - b^4\end{aligned}$$

- Or you can use FOIL to multiply out, collect like terms, and simplify:

$$\begin{aligned}(a^{1/2} - b^2)(a^{1/2} + b^2) &= a^{1/2} \cdot a^{1/2} + a^{1/2} \cdot b^2 - b^2 \cdot a^{1/2} - b^2 \cdot b^2 \\ &= a^{1/2+1/2} - b^{2+2} \\ &= a - b^4\end{aligned}$$

(b) $(1 - 2y)(1 + 2y + 4y^2)$

Again, there are 2 ways to do this problem:

- You can notice that it is in the form of “difference of cubes”:

$$\begin{aligned}(1 - 2y)(1 + 2y + 4y^2) &= 1^3 - (2y)^3 \\ &= 1 - 2^3 y^3 \\ &= 1 - 8y^3\end{aligned}$$

- Or you can multiply it out term by term (like FOIL but with more terms), collect like terms, and simplify:

$$\begin{aligned}(1 - 2y)(1 + 2y + 4y^2) &= 1 \cdot (1 + 2y + 4y^2) - 2y \cdot (1 + 2y + 4y^2) \\ &= (1 + 2y + 4y^2) - (2y + 4y^2 + 8y^3) \\ &= 1 + 2y + 4y^2 - 2y - 4y^2 - 8y^3 \\ &= 1 + (2y - 2y) + (4y^2 - 4y^2) - 8y^3 \\ &= 1 + 0 + 0 - 8y^3 \\ &= 1 - 8y^3\end{aligned}$$

2. Factor the following expressions completely:

(a) $24x^3 - 54x = 6x(4x^2 - 9) = 6x(2x - 3)(2x + 3)$

(b) $2x^3 + x^2 - 6x - 3 = (2x^3 + x^2) + (-6x - 3) = x^2(2x + 1) - 3(2x + 1) = (x^2 - 3)(2x + 1)$

(c) $(a - 1)^2(a + b)^3 + (a - 1)^3(a + b)^2$

$$\begin{aligned}(a - 1)^2(a + b)^3 + (a - 1)^3(a + b)^2 &= (a - 1)^2(a + b)^2(a + b) + (a - 1)^2(a - 1)(a + b)^2 \\ &= (a - 1)^2(a + b)^2(a + b) + (a - 1)^2(a + b)^2(a - 1) \\ &= [(a - 1)^2(a + b)^2][(a + b) + (a - 1)] \\ &= (a - 1)^2(a + b)^2[a + b + a - 1] \\ &= (a - 1)^2(a + b)^2(2a + b - 1)\end{aligned}$$