

1. Simplify each of the following by performing the indicated operation(s).

(a)  $(x^{3/2} - z)(x^{3/2} + z)$

There are 2 ways to do this problem:

- You can notice that it is in the form of “difference of squares”:

$$\begin{aligned}(x^{3/2} - z)(x^{3/2} + z) &= (x^{3/2})^2 - (z)^2 \\ &= x^{(3/2) \cdot 2} - z^2 \\ &= x^3 - z^2\end{aligned}$$

- Or you can use FOIL to multiply out, collect like terms, and simplify:

$$\begin{aligned}(x^{3/2} - z)(x^{3/2} + z) &= x^{3/2} \cdot x^{3/2} + x^{3/2} \cdot z - z \cdot x^{3/2} - z \cdot z \\ &= x^{3/2+3/2} - z^{1+1} \\ &= x^3 - z^2\end{aligned}$$

(b)  $(2 - x)(x^2 + 2x + 4)$

Again, there are 2 ways to do this problem:

- You can notice that it is in the form of “difference of cubes”:

$$\begin{aligned}(2 - x)(x^2 + 2x + 4) &= 2^3 - (x)^3 \\ &= 8 - x^3\end{aligned}$$

- Or you can multiply it out term by term (like FOIL but with more terms), collect like terms, and simplify:

$$\begin{aligned}(2 - x)(x^2 + 2x + 4) &= 2 \cdot (x^2 + 2x + 4) - x \cdot (x^2 + 2x + 4) \\ &= (2x^2 + 4x + 8) - (x^3 + 2x^2 + 4x) \\ &= 2x^2 + 4x + 8 - x^3 - 2x^2 - 4x \\ &= 8 + (4x - 4x) + (2x^2 - 2x^2) - x^3 \\ &= 8 + 0 + 0 - x^3 \\ &= 8 - x^3\end{aligned}$$

2. Factor the following expressions completely:

(a)  $4x^3 - 100x = 4x(x^2 - 25) = 4x(x - 5)(x + 5)$

(b)  $4x^3 + 2x^2 - 6x - 3 = (4x^3 + 2x^2) + (-6x - 3) = 2x^2(2x + 1) - 3(2x + 1) = (2x^2 - 3)(2x + 1)$

(c)  $(x + y)^2(x - 3) - (x + y)(x - 3)^2$

$$\begin{aligned}(x + y)^2(x - 3) - (x + y)(x - 3)^2 &= (x + y)(x + y)(x - 3) - (x + y)(x - 3)(x - 3) \\ &= (x + y)(x - 3)(x + y) - (x + y)(x - 3)(x - 3) \\ &= [(x + y)(x - 3)][(x + y) - (x - 3)] \\ &= (x + y)(x - 3)[x + y - x + 3] \\ &= (x + y)(x - 3)(y + 3)\end{aligned}$$