

1. Simplify each of the following by performing the indicated operation(s).

(a) $(a^2 - b^{3/2})(a^2 + b^{3/2})$

There are 2 ways to do this problem:

- You can notice that it is in the form of “difference of squares”:

$$\begin{aligned}(a^2 - b^{3/2})(a^2 + b^{3/2}) &= (a^2)^2 - (b^{3/2})^2 \\ &= a^{2 \cdot 2} - b^{(3/2) \cdot 2} \\ &= a^4 - b^3\end{aligned}$$

- Or you can use FOIL to multiply out, collect like terms, and simplify:

$$\begin{aligned}(a^2 - b^{3/2})(a^2 + b^{3/2}) &= a^2 \cdot a^2 + a^2 \cdot b^{3/2} - b^{3/2} \cdot a^2 - b^{3/2} \cdot b^{3/2} \\ &= a^{2+2} - b^{3/2+3/2} \\ &= a^4 - b^3\end{aligned}$$

(b) $(3y - 2)(9y^2 + 6y + 4)$

Again, there are 2 ways to do this problem:

- You can notice that it is in the form of “difference of cubes”:

$$\begin{aligned}(3y - 2)(9y^2 + 6y + 4) &= (3y)^3 - (2)^3 \\ &= 3^3 y^3 - 8 \\ &= 27y^3 - 8\end{aligned}$$

- Or you can multiply it out term by term (like FOIL but with more terms), collect like terms, and simplify:

$$\begin{aligned}(3y - 2)(9y^2 + 6y + 4) &= 3y \cdot (9y^2 + 6y + 4) - 2 \cdot (9y^2 + 6y + 4) \\ &= (27y^3 + 18y^2 + 12y) - (18y^2 + 12y + 8) \\ &= 27y^3 + 18y^2 + 12y - 18y^2 - 12y - 8 \\ &= 27y^3 + (18y^2 - 18y^2) + (12y - 12y) - 8 \\ &= 27y^3 + 0 + 0 - 8 \\ &= 27y^3 - 8\end{aligned}$$

2. Factor the following expressions completely:

(a) $4x^3 - 36x = 4x(x^2 - 9) = 4x(x - 3)(x + 3)$

(b) $2x^3 + 8x^2 - x - 4 = (2x^3 + 8x^2) + (-x - 4) = 2x^2(x + 4) - (x + 4) = (2x^2 - 1)(x + 4)$

(c) $(x - 3)^2(y - 4)^3 + (y - 4)^2(x - 3)^3$

$$\begin{aligned}(x - 3)^2(y - 4)^3 + (y - 4)^2(x - 3)^3 &= (x - 3)^2(y - 4)^2(y - 4) + (y - 4)^2(x - 3)^2(x - 3) \\ &= (x - 3)^2(y - 4)^2(y - 4) + (x - 3)^2(y - 4)^2(x - 3) \\ &= [(x - 3)^2(y - 4)^2][(y - 4) + (x - 3)] \\ &= (x - 3)^2(y - 4)^2[y - 4 + x - 3] \\ &= (x - 3)^2(y - 4)^2(y + x - 7)\end{aligned}$$