

1. Simplify each of the following completely.

$$(a) \frac{2x^2 - 3x - 20}{2x^2 - 9x - 35} \cdot \frac{x^2 - 3x - 28}{x^2 + 3x - 28}$$

First, factor each quadratic:

- $(2x^2 - 3x - 20) = (2x + 5)(x - 4)$
- $(2x^2 - 9x - 35) = (2x + 5)(x - 7)$
- $(x^2 - 3x - 28) = (x + 4)(x - 7)$
- $(x^2 + 3x - 28) = (x - 4)(x + 7)$.

$$\begin{aligned} \frac{2x^2 - 3x - 20}{2x^2 - 9x - 35} \cdot \frac{x^2 - 3x - 28}{x^2 + 3x - 28} &= \frac{(2x + 5)(x - 4)}{(2x + 5)(x - 7)} \cdot \frac{(x + 4)(x - 7)}{(x - 4)(x + 7)} \\ &= \frac{x + 4}{x + 7} \end{aligned}$$

$$(b) \frac{\frac{7}{x-2} + \frac{x}{x+2}}{\frac{1}{x+2} - \frac{3}{x-2}}$$

First, find the common denominator for the top and the bottom:

- $\frac{7}{x-2} + \frac{x}{x+2} = \frac{7(x+2)}{(x-2)(x+2)} + \frac{x(x-2)}{(x+2)(x-2)} = \frac{7x+14+x^2-2x}{(x+2)(x-2)}$
- $\frac{1}{x+2} - \frac{3}{x-2} = \frac{(x-2)}{(x+2)(x-2)} - \frac{3(x+2)}{(x-2)(x+2)} = \frac{x-2-3x-6}{(x-2)(x+2)}$

$$\begin{aligned} \frac{\frac{7}{x-2} + \frac{x}{x+2}}{\frac{1}{x+2} - \frac{3}{x-2}} &= \frac{\frac{7x+14+x^2-2x}{(x+2)(x-2)}}{\frac{x-2-3x-6}{(x-2)(x+2)}} \\ &= \frac{x^2+5x+14}{(x+2)(x-2)} \cdot \frac{(x-2)(x+2)}{-2x-8} \\ &= \frac{x^2+5x+14}{-2x-8} \end{aligned}$$

2. Solve each of the following equations.

$$(a) 2x^2 - 3x - 1 = 0$$

Use the quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case, $a = 2, b = -3, c = -1$.

$$\begin{aligned}x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-1)}}{2(2)} \\&= \frac{3 \pm \sqrt{9+8}}{4} \\&= \frac{3 \pm \sqrt{17}}{4}\end{aligned}$$

$$(b) \frac{1}{y-1} + \frac{3}{y-2} = \frac{5}{y-1}$$

$$\begin{aligned}\frac{1}{y-1} + \frac{3}{y-2} &= \frac{5}{y-1} \\ \frac{(y-2)}{(y-1)(y-2)} + \frac{3(y-1)}{(y-2)(y-1)} &= \frac{5(y-2)}{(y-1)(y-2)} \\ \frac{y-2+3y-3}{(y-1)(y-2)} &= \frac{5y-10}{(y-1)(y-2)} \\ 4y-5 &= 5y-10 \\ y &= -5\end{aligned}$$

$$(c) S = \frac{a}{1-r}; \text{ for } r$$

$$\begin{aligned}S &= \frac{a}{1-r} \\ S(1-r) &= a \\ 1-r &= \frac{a}{S} \\ -r &= \frac{a}{S} - 1 \\ r &= -\frac{a}{S} + 1\end{aligned}$$