

1. Simplify each of the following completely.

$$(a) \frac{2x^2 - 7x - 15}{2x^2 + 13x + 15} \cdot \frac{x^2 - x - 30}{x^2 + x - 30}$$

First, factor each quadratic:

- $(2x^2 - 7x - 15) = (2x + 3)(x - 5)$
- $(2x^2 + 13x + 15) = (2x + 3)(x + 5)$
- $(x^2 - x - 30) = (x + 5)(x - 6)$
- $(x^2 + x - 30) = (x + 6)(x - 5)$.

$$\begin{aligned} \frac{2x^2 - 7x - 15}{2x^2 + 13x + 15} \cdot \frac{x^2 - x - 30}{x^2 + x - 30} &= \frac{(2x + 3)(x - 5)}{(2x + 3)(x + 5)} \cdot \frac{(x + 5)(x - 6)}{(x + 6)(x - 5)} \\ &= \frac{x - 6}{x + 6} \end{aligned}$$

$$(b) \frac{\frac{5}{x-3} + \frac{2x}{x+3}}{\frac{x+3}{x+3} - \frac{x-3}{x-3}}$$

First, find the common denominator for the top and the bottom:

- $\frac{5}{x-3} + \frac{2x}{x+3} = \frac{5(x+3)}{(x-3)(x+3)} + \frac{2x(x-3)}{(x+3)(x-3)} = \frac{5x+15+2x^2-6x}{(x+3)(x-3)}$
- $\frac{2}{x+3} - \frac{7}{x-3} = \frac{2(x-3)}{(x+3)(x-3)} - \frac{7(x+3)}{(x-3)(x+3)} = \frac{2x-6-7x-21}{(x-3)(x+3)}$

$$\begin{aligned} \frac{\frac{5}{x-3} + \frac{2x}{x+3}}{\frac{x+3}{x+3} - \frac{x-3}{x-3}} &= \frac{\frac{5x+15+2x^2-6x}{(x+3)(x-3)}}{\frac{2x-6-7x-21}{(x-3)(x+3)}} \\ &= \frac{2x^2-x+15}{(x+3)(x-3)} \cdot \frac{(x-3)(x+3)}{-5x-27} \\ &= \frac{2x^2-x+15}{-5x-27} \end{aligned}$$

2. Solve each of the following equations.

$$(a) 3x^2 - 4x - 1 = 0$$

Use the quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case, $a = 3$, $b = -4$, $c = -1$.

$$\begin{aligned}
 x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-1)}}{2(3)} \\
 &= \frac{4 \pm \sqrt{16 + 12}}{6} \\
 &= \frac{4 \pm \sqrt{28}}{6} \\
 &= \frac{4 \pm \sqrt{4 \cdot 7}}{6} \\
 &= \frac{4 \pm 2\sqrt{7}}{6} \\
 &= \frac{2 \pm \sqrt{7}}{3}
 \end{aligned}$$

$$(b) \quad \frac{5}{y-1} + \frac{3}{y-2} = \frac{1}{y-1}$$

$$\begin{aligned}
 \frac{5}{y-1} + \frac{3}{y-2} &= \frac{1}{y-1} \\
 \frac{5(y-2)}{(y-1)(y-2)} + \frac{3(y-1)}{(y-2)(y-1)} &= \frac{(y-2)}{(y-1)(y-2)} \\
 \frac{5y-10+3y-3}{(y-1)(y-2)} &= \frac{y-2}{(y-1)(y-2)} \\
 8y-13 &= y-2 \\
 7y &= 11 \\
 y &= \frac{11}{7}
 \end{aligned}$$

$$(c) \quad S = \frac{a}{1-r}; \text{ for } r$$

$$\begin{aligned}
 S &= \frac{a}{1-r} \\
 S(1-r) &= a \\
 1-r &= \frac{a}{S} \\
 -r &= \frac{a}{S} - 1 \\
 r &= -\frac{a}{S} + 1
 \end{aligned}$$