

1. Simplify each of the following completely.

$$(a) \frac{2x^2 - 5x - 12}{2x^2 + 11x + 12} \cdot \frac{x^2 - 2x - 24}{x^2 + 2x - 24}$$

First, factor each quadratic:

- $(2x^2 - 5x - 12) = (2x + 3)(x - 4)$
- $(2x^2 + 11x + 12) = (2x + 3)(x + 4)$
- $(x^2 - 2x - 24) = (x + 4)(x - 6)$
- $(x^2 + 2x - 24) = (x + 6)(x - 4)$.

$$\begin{aligned} \frac{2x^2 - 5x - 12}{2x^2 + 11x + 12} \cdot \frac{x^2 - 2x - 24}{x^2 + 2x - 24} &= \frac{(2x + 3)(x - 4)}{(2x + 3)(x + 4)} \cdot \frac{(x + 4)(x - 6)}{(x + 6)(x - 4)} \\ &= \frac{x - 6}{x + 6} \end{aligned}$$

$$(b) \frac{\frac{5}{x-4} + \frac{2x}{x+4}}{\frac{1}{x+4} - \frac{6}{x-4}}$$

First, find the common denominator for the top and the bottom:

- $\frac{5}{x-4} + \frac{2x}{x+4} = \frac{5(x+4)}{(x-4)(x+4)} + \frac{2x(x-4)}{(x+4)(x-4)} = \frac{5x+20+2x^2-8x}{(x+4)(x-4)}$
- $\frac{1}{x+4} - \frac{6}{x-4} = \frac{(x-4)}{(x+4)(x-4)} - \frac{6(x+4)}{(x-4)(x+4)} = \frac{x-4-6x-24}{(x-4)(x+4)}$

$$\begin{aligned} \frac{\frac{5}{x-4} + \frac{2x}{x+4}}{\frac{1}{x+4} - \frac{6}{x-4}} &= \frac{\frac{5x+20+2x^2-8x}{(x+4)(x-4)}}{\frac{x-4-6x-24}{(x-4)(x+4)}} \\ &= \frac{2x^2-3x+20}{(x+4)(x-4)} \cdot \frac{(x-4)(x+4)}{-5x-28} \\ &= \frac{2x^2-3x+20}{-5x-28} \end{aligned}$$

2. Solve each of the following equations.

$$(a) 4x^2 - 3x - 2 = 0$$

Use the quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case, $a = 4$, $b = -3$, $c = -2$.

$$\begin{aligned}x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-2)}}{2(4)} \\&= \frac{3 \pm \sqrt{9 + 32}}{8} \\&= \frac{3 \pm \sqrt{41}}{8}\end{aligned}$$

$$(b) \quad \frac{2}{y-3} + \frac{4}{y-1} = \frac{5}{y-1}$$

$$\begin{aligned}\frac{2}{y-3} + \frac{4}{y-1} &= \frac{5}{y-1} \\ \frac{2(y-1)}{(y-3)(y-1)} + \frac{4(y-3)}{(y-1)(y-3)} &= \frac{5(y-3)}{(y-1)(y-3)} \\ \frac{2y-2+4y-12}{(y-1)(y-3)} &= \frac{5y-15}{(y-1)(y-3)} \\ 6y-14 &= 5y-15 \\ y &= -1\end{aligned}$$

$$(c) \quad S = \frac{a}{1-r}; \text{ for } r$$

$$\begin{aligned}S &= \frac{a}{1-r} \\ S(1-r) &= a \\ 1-r &= \frac{a}{S} \\ -r &= \frac{a}{S} - 1 \\ r &= -\frac{a}{S} + 1\end{aligned}$$