1. Find an explicit solution $y(x)$ of the initial value problem

$$y \cos x + 1 + (\sin x + 1) \frac{dy}{dx} = 0, \quad y(0) = 3.$$

**Method 1 (intended):** Use the fact that this is exact.

Method: $M = y \cos x + 1, \quad N = \sin x + 1 \quad \checkmark$

$$\frac{\partial M}{\partial y} = M, \quad \frac{\partial N}{\partial x} = N$$

$$\frac{\partial M}{\partial y} = y \cos x + 1 \quad \text{and} \quad \frac{\partial N}{\partial x} = \sin x + 1$$

Because we are setting $\Psi(x,y)$ equal to a constant, we can suppress the extra "$+c$" that would go here.

General Solution: $y \sin x + x + 3 = C$

Initial Value: $y(0) = 3$

$3 \cdot \sin(0) + 0 + 3 = C \implies C = 3$

Final Solution: $y \sin x + x + 3 = 3$

**Method 2: Integrating Factors**

Rewrite: $y' + \frac{\cos(x)}{\sin(x) + 1} \cdot y = \frac{-1}{\sin(x) + 1}$

$$\left(\sin(x) + 1\right) y' + \cos(x) \cdot y = -1$$

$$\frac{d}{dx} \left[ (\sin(x) + 1) y \right] = -1$$

Integrate wrt $x$...

$$(\sin(x) + 1) y = -x + C$$

$$y = \frac{-x + C}{\sin(x) + 1}$$

Initial Value: $y(0) = 3$

$$3 = \frac{-0 + C}{1} \implies C = 3$$

Final Solution: $y = \frac{-x + 3}{\sin(x) + 1}$
2. Consider the problem \( \frac{dy}{dt} - \frac{1}{\cos t} y = \ln|t - 1|, \ y(0) = 2 \). **Without solving the problem**, determine the largest interval on which the solution is guaranteed to exist.

**Theorem:** Whenever \( P(t) \), \( G(t) \) are continuous, the solution is guaranteed to exist (this is a slight paraphrase),

\[ P(t) = \frac{-1}{\cos(t)}: \text{ not continuous when } \cos(t) = 0, \ i.e. \]

\( t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots \) and \( -\frac{\pi}{2}, -\frac{3\pi}{2} \ldots \)

\[ G(t) = \ln|t - 1|: \text{ not continuous when } |t - 1| \leq 0. \]

Since there is an absolute value, the only problem is when \( t - 1 = 0 \), or \( t = -1 \).

**Bad Places:**

Our solution to the initial value problem \( y(0) = 2 \) contains the point \( t = 0 \). The largest interval where this is guaranteed to exist is \( (-1, \frac{\pi}{2}) \).

3. A certain variable \( N(t) \), taking values \( -\infty < N < \infty \), satisfies \( \frac{dN}{dt} = N(N^2 - 1) \).

Find the equilibrium (critical) values of \( N \) and classify each as stable or unstable.

**Critical values occur when** \( \frac{dN}{dt} = 0 \), or \( N(N^2 - 1) = 0 \).

**Critical values:** \( N = 0, 1, -1 \).

Unstable equilibrium at \( N = 1 \): solutions slightly above or below tend away from equilibrium value.

Stable equilibrium at \( N = 0 \): solutions slightly above or below tend towards equilibrium value.

Unstable equilibrium at \( N = -1 \).