2. Consider the problem \( \frac{dy}{dt} - \frac{1}{\cos t} y = \ln|t - 1| \), \( y(0) = 2 \). Without solving the problem, determine the largest interval on which the solution is guaranteed to exist.

Theorem: Whenever \( P(t) \), \( G(t) \) are continuous, the solution is guaranteed to exist (this is a slight paraphrase).

\[ P(t) = \frac{-1}{\cos(t)} ; \text{not continuous when } \cos(t) = 0, \text{ i.e.} \]
\[ t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots, \text{ and } -\frac{\pi}{2}, -\frac{3\pi}{2} \ldots \]

\[ G(t) = \ln|t-1| ; \text{not continuous when } |t-1| \leq 0. \]

Since there is an absolute value, the only problem is when \( t-1 = 0 \), or \( t = 1 \).

**Bad Places:**

Our solution to the initial value problem \( y(0) = 2 \) contains the point \( t = 0 \), the largest interval where this is guaranteed to exist is \( (-1, \frac{\pi}{2}) \).

3. A certain variable \( N(t) \), taking values \(-\infty < N < \infty\), satisfies \( dN/dt = N(N^2 - 1) \).

Find the equilibrium (critical) values of \( N \) and classify each as stable or unstable.

**Critical Values** occur when \( dN/dt = 0 \), or \( N(N^2 - 1) = 0 \).

Critical values: \( N = 0, 1, -1 \).

Unstable equilibrium at \( N = 1 \): solutions slightly above or below tend away from equilibrium value.

Stable equilibrium at \( N = 0 \): solutions slightly above or below tend towards equilibrium value.

Unstable equilibrium at \( N = -1 \).
1. Find an explicit solution \( y(x) \) of the initial value problem

\[
y \cos x + 1 + (\sin x + 1) \frac{dy}{dx} = 0, \quad y(0) = 3.
\]

**Method 1 (intended):** Use the fact that this is exact.

\[
M = y \cos x + 1, \quad M_y = \cos x \\
N = \sin x + 1, \quad N_x = \cos x
\]

Because we are setting \( \Psi(x, y) \) equal to a constant, we can suppress the extra "+ C" that would go here.

General Solution: \( y \sin x + x + h = C \)

Initial Value: \( y(0) = 3 \)

\[
3 \cdot \sin 0 + 0 + 3 = C \implies C = 3
\]

**Method 2: Integrating Factors**

Rewrite: \( y' + \frac{\cos x}{\sin x + 1} y = \frac{-1}{\sin x + 1} \)

\[
\frac{\sin (x+1)}{\sin (x+1)} y' + \cos (x) \cdot \frac{dy}{dx} = -1
\]

\[
\frac{d}{dx} \left[ (\sin (x+1)) y \right] = -1
\]

Integrate wrt \( x \):

\[
(\sin (x+1)) y = -x + C
\]

Initial Value: \( y(0) = 3 \)

\[
3 = \frac{0 + C}{1} \implies C = 3
\]

Final Solution: \( y = \frac{-x + 3}{\sin (x+1)} \)