

**read** "/Users/bethkupin/Desktop/GeneralizedGouldenJackson.txt";

*This is the package GeneralizedGouldenJackson, written by Beth Kupin and Debbie Yuster* (1)

Two examples, using SingleGJ to compute generating functions:

*SingleGJ*( {a, b}, {[a, b], [b, a]}, t, X);

$$-\frac{-1 + t^2 X_b X_a}{1 + t^2 X_b X_a - t X_a - t X_b} \quad (2)$$

*normal*(*taylor*(*SingleGJ*( {a, b}, {[a, b], [b, a]}, t, X), t, 5));

$$1 + (X_a + X_b) t + (X_a^2 + X_b^2) t^2 + (X_a^3 + X_b^3) t^3 + (X_a^4 + X_b^4) t^4 + O(t^5) \quad (3)$$

*SingleGJ*( {a, b}, {[a, b], [b, b, a]}, t, X);

$$-\frac{-1 + t^3 X_b^2 X_a}{1 + t^2 X_b X_a - t X_a - t X_b} \quad (4)$$

*normal*(*taylor*(*SingleGJ*( {a, b}, {[a, b], [b, b, a]}, t, X), t, 5));

$$1 + (X_a + X_b) t + (X_b X_a + X_a^2 + X_b^2) t^2 + (X_b X_a^2 + X_a^3 + X_b^3) t^3 + (X_b X_a^3 + X_a^4 + X_b^4) t^4 + O(t^5) \quad (5)$$

The same two examples as above, this time with DoubleGJ and ProbDoubleGJ, so that you can see the alteration in how the generating function appears.

*DoubleGJ*( {a, b}, {[a, b], [b, a]}, t, X);

$$\frac{1}{(-1 + t X_{ID, a} X_{a, a}) (-1 + t X_{ID, b} X_{b, b})} \left( 1 - t X_{ID, b} X_{b, b} - t X_{ID, a} X_{a, a} + t^2 X_{ID, a} X_{a, a} X_{ID, b} X_{b, b} + t X_{ID, a} - t^2 X_{ID, a} X_{ID, b} X_{b, b} + t X_{ID, b} - t^2 X_{ID, b} X_{ID, a} X_{a, a} \right) \quad (6)$$

*normal*(*taylor*(*DoubleGJ*( {a, b}, {[a, b], [b, a]}, t, X), t, 5));

$$1 + (X_{ID, a} + X_{ID, b}) t + (X_{ID, a}^2 X_{a, a} + X_{ID, b}^2 X_{b, b}) t^2 + (X_{ID, a}^3 X_{a, a} + X_{ID, b}^3 X_{b, b}) t^3 + (X_{ID, a}^4 X_{a, a} + X_{ID, b}^4 X_{b, b}) t^4 + O(t^5) \quad (7)$$

*DoubleGJ*( {a, b}, {[a, b], [b, b, a]}, t, X);

$$\frac{1}{(-1 + t X_{ID, b} X_{b, b}) (-1 + t X_{ID, a} X_{a, a})} \left( 1 - t X_{ID, a} X_{a, a} - t X_{ID, b} X_{b, b} + t^2 X_{ID, a} X_{a, a} X_{ID, b} X_{b, b} + t X_{ID, a} - t^2 X_{ID, a} X_{ID, b} X_{b, b} + t^2 X_{ID, b} X_{ID, a} X_{b, a} - t^2 X_{ID, b} X_{ID, a} X_{a, a} + t X_{ID, b} - t^3 X_{ID, b}^2 X_{ID, a} X_{b, b} X_{b, a} \right) \quad (8)$$

*normal*(*taylor*(*DoubleGJ*( {a, b}, {[a, b], [b, b, a]}, t, X), t, 5));

$$\begin{aligned}
& 1 + (X_{ID,a} + X_{ID,b})t + (X_{ID,a}X_{b,a}X_{ID,b} + X_{ID,b}^2X_{b,b} + X_{ID,a}^2X_{a,a})t^2 + (X_{ID,b}^3X_{b,b}^2 + \\
& X_{ID,a}^2X_{a,a}X_{b,a}X_{ID,b} + X_{ID,a}^3X_{a,a}^2)t^3 + (X_{ID,b}^4X_{b,b}^3 + X_{ID,a}^3X_{a,a}^2X_{b,a}X_{ID,b} + X_{ID,a}^4X_{a,a}^3)t^4 \\
& + O(t^5)
\end{aligned} \tag{9}$$

$ProbDoubleGJ(\{a, b\}, \{[a, b], [b, a]\}, t, X);$

$$-\frac{-1 + tX_{b,b} + tX_{a,a} - t^2X_{a,a}X_{b,b} - tX_{ID,a} + t^2X_{ID,a}X_{b,b} - tX_{ID,b} + t^2X_{ID,b}X_{a,a}}{(-1 + tX_{a,a})(-1 + tX_{b,b})} \tag{10}$$

$normal(taylor(ProbDoubleGJ(\{a, b\}, \{[a, b], [b, a]\}, t, X), t, 5));$

$$\begin{aligned}
& 1 + (X_{ID,a} + X_{ID,b})t + (X_{a,a}X_{ID,a} + X_{b,b}X_{ID,b})t^2 + (X_{a,a}^2X_{ID,a} + X_{b,b}^2X_{ID,b})t^3 + ( \\
& X_{a,a}^3X_{ID,a} + X_{b,b}^3X_{ID,b})t^4 + O(t^5)
\end{aligned} \tag{11}$$

$ProbDoubleGJ(\{a, b\}, \{[a, b], [b, b, a]\}, t, X);$

$$\begin{aligned}
& -\frac{1}{(-1 + tX_{a,a})(-1 + tX_{b,b})} \left( -1 + tX_{b,b} + tX_{a,a} - t^2X_{a,a}X_{b,b} - tX_{ID,a} + t^2X_{ID,a}X_{b,b} \right. \\
& \left. - t^2X_{ID,b}X_{b,a} + t^2X_{ID,b}X_{a,a} - tX_{ID,b} + t^3X_{ID,b}X_{b,b}X_{b,a} \right)
\end{aligned} \tag{12}$$

$normal(taylor(ProbDoubleGJ(\{a, b\}, \{[a, b], [b, b, a]\}, t, X), t, 5));$

$$\begin{aligned}
& 1 + (X_{ID,a} + X_{ID,b})t + (X_{ID,b}X_{b,a} + X_{a,a}X_{ID,a} + X_{b,b}X_{ID,b})t^2 + (X_{a,a}X_{ID,b}X_{b,a} + \\
& X_{a,a}^2X_{ID,a} + X_{b,b}^2X_{ID,b})t^3 + (X_{a,a}^2X_{ID,b}X_{b,a} + X_{a,a}^3X_{ID,a} + X_{b,b}^3X_{ID,b})t^4 + O(t^5)
\end{aligned} \tag{13}$$

The same two examples, now in TripleGJ. Note that the degree and number of terms of the numerator of the generating function has changed, as the structure of the program becomes more complicated.

$TripleGJ(\{a, b\}, \{[a, b], [b, a]\}, t, X);$

$$\begin{aligned}
& (1 - tX_{ID,ID,b}X_{b,b,b}X_{ID,b,b} - tX_{ID,ID,a}X_{a,a,a}X_{ID,a,a} \\
& + t^2X_{ID,ID,a}X_{a,a,a}X_{ID,a,a}X_{ID,ID,b}X_{b,b,b}X_{ID,b,b} + tX_{ID,ID,a} \\
& - X_{ID,ID,a}t^2X_{ID,ID,b}X_{b,b,b}X_{ID,b,b} - t^2X_{ID,ID,a}^2X_{a,a,a}X_{ID,a,a} + t^3 \\
& X_{ID,ID,a}^2X_{a,a,a}X_{ID,a,a}X_{ID,ID,b}X_{b,b,b}X_{ID,b,b} + tX_{ID,ID,b} - t^2X_{ID,ID,b}^2X_{b,b,b}X_{ID,b,b} \\
& - X_{ID,ID,b}t^2X_{ID,ID,a}X_{a,a,a}X_{ID,a,a} + X_{ID,ID,b}^2t^3X_{ID,ID,a}X_{a,a,a}X_{ID,a,a}X_{b,b,b}X_{ID,b,b} \\
& + X_{ID,a,a}X_{ID,ID,a}^2t^2 - X_{ID,a,a}X_{ID,ID,a}^2t^3X_{ID,ID,b}X_{b,b,b}X_{ID,b,b} + X_{ID,b,b}X_{ID,ID,b}^2t^2 \\
& - X_{ID,b,b}X_{ID,ID,b}^2t^3X_{ID,ID,a}X_{a,a,a}X_{ID,a,a}) / ((-1 + tX_{ID,ID,a}X_{a,a,a}X_{ID,a,a})(-1 \\
& + tX_{ID,ID,b}X_{b,b,b}X_{ID,b,b}))
\end{aligned} \tag{14}$$

$normal(taylor(TripleGJ(\{a, b\}, \{[a, b], [b, a]\}, t, X), t, 5));$

$$\begin{aligned}
& 1 + (X_{ID, ID, a} + X_{ID, ID, b}) t + (X_{ID, a, a} X_{ID, ID, a}^2 + X_{ID, b, b} X_{ID, ID, b}^2) t^2 + (X_{ID, ID, a}^3 X_{a, a, a} \\
& \quad X_{ID, a, a}^2 + X_{ID, ID, b}^3 X_{b, b, b} X_{ID, b, b}^2) t^3 + (X_{ID, ID, a}^4 X_{a, a, a}^2 X_{ID, a, a}^3 + X_{ID, ID, b}^4 X_{b, b, b}^2 X_{ID, b, b}^3) t^4 \\
& \quad + O(t^5)
\end{aligned} \tag{15}$$

$TripleGJ(\{a, b\}, \{[a, b], [b, b, a]\}, t, X);$

$$\begin{aligned}
& - \left( -1 + t X_{ID, ID, b} X_{b, b, b} X_{ID, b, b} + t X_{ID, ID, a} X_{a, a, a} X_{ID, a, a} \right. \\
& \quad - t^2 X_{ID, ID, a} X_{a, a, a} X_{ID, a, a} X_{ID, ID, b} X_{b, b, b} X_{ID, b, b} - t X_{ID, ID, a} \\
& \quad + X_{ID, ID, a} t^2 X_{ID, ID, b} X_{b, b, b} X_{ID, b, b} + t^2 X_{ID, ID, a}^2 X_{a, a, a} X_{ID, a, a} - t^3 \\
& \quad X_{ID, ID, a}^2 X_{a, a, a} X_{ID, a, a} X_{ID, ID, b} X_{b, b, b} X_{ID, b, b} - t X_{ID, ID, b} + t^2 X_{ID, ID, b}^2 X_{b, b, b} X_{ID, b, b} \\
& \quad + X_{ID, ID, b} t^2 X_{ID, ID, a} X_{a, a, a} X_{ID, a, a} - X_{ID, ID, b}^2 t^3 X_{ID, ID, a} X_{a, a, a} X_{ID, a, a} X_{b, b, b} X_{ID, b, b} \\
& \quad - X_{ID, a, a} X_{ID, ID, a}^2 t^2 + X_{ID, a, a} X_{ID, ID, a}^2 t^3 X_{ID, ID, b} X_{b, b, b} X_{ID, b, b} - X_{ID, b, a} X_{ID, ID, b} \\
& \quad X_{ID, ID, a}^2 t^3 X_{ID, a, a} X_{b, a, a} + X_{ID, b, a} X_{ID, ID, b}^2 X_{ID, ID, a}^2 t^4 X_{ID, a, a} X_{b, a, a} X_{b, b, b} X_{ID, b, b} \\
& \quad + X_{ID, b, a} X_{ID, ID, b} X_{ID, ID, a}^2 t^3 X_{a, a, a} X_{ID, a, a} - X_{ID, b, a} X_{ID, ID, b}^2 \\
& \quad X_{ID, ID, a}^2 t^4 X_{a, a, a} X_{ID, a, a} X_{b, b, b} X_{ID, b, b} - X_{ID, b, a} X_{ID, ID, b} X_{ID, ID, a} t^2 + X_{ID, b, a} \\
& \quad X_{ID, ID, b}^2 X_{ID, ID, a} t^3 X_{b, b, b} X_{ID, b, b} - X_{ID, b, b} X_{ID, ID, b}^2 t^2 + X_{ID, b, b} \\
& \quad \left. X_{ID, ID, b}^2 t^3 X_{ID, ID, a} X_{a, a, a} X_{ID, a, a} \right) / \left( (-1 + t X_{ID, ID, a} X_{a, a, a} X_{ID, a, a}) (-1 \right. \\
& \quad \left. + t X_{ID, ID, b} X_{b, b, b} X_{ID, b, b}) \right)
\end{aligned} \tag{16}$$

$normal(taylor(TripleGJ(\{a, b\}, \{[a, b], [b, b, a]\}, t, X), t, 5));$

$$\begin{aligned}
& 1 + (X_{ID, ID, a} + X_{ID, ID, b}) t + (X_{ID, b, b} X_{ID, ID, b}^2 + X_{ID, a, a} X_{ID, ID, a}^2 \\
& \quad + X_{ID, b, a} X_{ID, ID, b} X_{ID, ID, a}) t^2 + (X_{ID, b, a} X_{ID, ID, b} X_{ID, ID, a}^2 X_{ID, a, a} X_{b, a, a} + \\
& \quad X_{ID, ID, a}^3 X_{a, a, a} X_{ID, a, a}^2 + X_{ID, ID, b}^3 X_{b, b, b} X_{ID, b, b}^2) t^3 + (X_{ID, ID, a}^3 X_{a, a, a} \\
& \quad X_{ID, a, a}^2 X_{ID, b, a} X_{ID, ID, b} X_{b, a, a} + X_{ID, ID, a}^4 X_{a, a, a}^2 X_{ID, a, a}^3 + X_{ID, ID, b}^4 X_{b, b, b}^2 X_{ID, b, b}^3) t^4 \\
& \quad + O(t^5)
\end{aligned} \tag{17}$$