

FORMULAS

Polynomial of degree $\leq n$ interpolating f at x_0, \dots, x_n

$$\text{Newton form : } P_n(x) = \sum_{i=0}^n f[x_0, x_1, \dots, x_i] \prod_{j=0}^{i-1} (x - x_j)$$

$$\text{Lagrange form : } P_n(x) = \sum_{j=0}^n L_{j,n}(x) f(x_j), \text{ where } L_{j,n}(x) = \prod_{\substack{i=0 \\ i \neq j}}^n \left(\frac{x - x_i}{x_j - x_i} \right).$$

$$\text{Error : } f(x) - P_n(x) = f[x_0, x_1, \dots, x_n, x] \prod_{i=0}^n (x - x_i) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \prod_{i=0}^n (x - x_i)$$

where

$$f[x_i, x_{i+1}, \dots, x_k] = \frac{f[x_{i+1}, \dots, x_k] - f[x_i, \dots, x_{k-1}]}{x_k - x_i}$$