Quiz 5, Math 441
Due Wed. Oct. 20, No late quiz will be accepted

Name – – – – – – – – – – – – – – – – – – $(Last)$ – – – – – – – – – – – – – – – $(First)$

Problem 1 (5pts). Prove that the standard topology in R^2 has a countable basis.

Problem. 2 (6pts). (a) Prove that the Zariski topology T (i.e., finite complement topology) on $\mathbf{R}^1$ is not derived from any metric d on $\mathbf{R}^1$. (Hint: show that any two non-empty open sets in T have non-empty intersection.)

(b) Consider R^1 as a subspace of R^2 (i.e., the x-axis). Let R^2 have the standard topology. Show that the subspace topology on R^1 is the standard topology.

Problem 3.(4pts) Let R^2 have the order topology T (with the product order). Let T' be the product topology $(R^1, T_d) \times (R^1, T_{st})$ where T_d and T_{st} be the discrete and the standard topologies. Is it true that $T = T'$? Justify your assertion.

Problem 4 (5pts). Let R^2 have the order topology T. Decide which of the following sets are open? Justify your assertions.

(a) A is the diagonal $\{(x, x) | x \in R^1\}$

(b) B is the half-plane $\{(x, y) | x \geq 0\}$

(c) C is the upper-half plane $\{(x, y) | y \geq 0\}$

(d) D $=\{(x, y) | y > x^2\}$.

(e) Find all lines L in the plane which are open sets.