

**Quiz 6, Math 441**  
**Due Wed. Oct. 27, No late quiz will be accepted**

Name ————— (Last) ————— (First)

Problem 1 (6pts). (a) Let the closure of a set  $V$  be  $cl(V)$ . Show that  $cl(A \cap B) \subset cl(A) \cap cl(B)$ .

(b) Show that every order topology is Hausdorff.

(c) Show that  $X$  is Hausdorff if and only if the diagonal  $D = \{(x, x) | x \in X\}$  is closed in  $X \times X$ .

Problem. 2 (4pts). Find all limit points and interior of the following subsets of  $\mathbf{R}^2$  with the standard topology. Justify your assertions.

(a)  $A = \mathbf{R} \times \{0\}$

(b)  $B = \{(x, y) \mid 1 < x^2 + y^2 \leq 2\}$

(c)  $C = B - A$

(d)  $D = \{(x, y) \mid |xy| \geq 1\}$ .

Problem 3.(5pts) Suppose  $f$  and  $g$  are two continuous maps from  $X \rightarrow \mathbf{R}$  where  $\mathbf{R}$  has the standard topology. Show that the set  $\{x \in X | f(x) \leq g(x)\}$  is closed. (hint: consider the map  $(f,g): X \rightarrow \mathbf{R}^2$ ).

Problem 4 (5pts). If  $f : Z \rightarrow Y$  is a map so that for any subset  $B \subset Z$ ,  $f(\bar{B}) \subset \bar{f(B)}$ , show that  $f$  is continuous.