## Homework 3

1. Find the general solution to the following ODEs

$$
\begin{aligned}
\frac{d y}{d x} & =\frac{x-e^{-x}}{y+e^{y}} \\
x y^{\prime} & =\sqrt{9-y^{2}}
\end{aligned}
$$

2. Find the solution to the following IVPs

$$
\begin{aligned}
y^{\prime} & =(1-2 x) y^{2}, y(0)=-1 / 6 \\
y^{\prime} & =\frac{x\left(x^{2}+1\right)}{4 y^{3}}, y(0)=-1 / \sqrt{2}
\end{aligned}
$$

3. Find the interval of existence for the following IVPs

$$
\begin{gathered}
y^{\prime}+\frac{t^{4}}{(t-2)^{8}} y=\sqrt{t}, y(1)=8 \\
\left(1-t^{4}\right) y^{\prime}+(\ln t) y=\cot 2 t, y(2)=0
\end{gathered}
$$

4. Find the region of $\left(x_{0}, y_{0}\right)$ in the $x y$-plane such that the following IVP is guaranteed to have at least a local solution

$$
\begin{gathered}
y^{\prime}=\frac{1}{1+2 y-3 t}, y\left(x_{0}\right)=y_{0} \\
y^{\prime}=\sqrt{y+t}, y\left(x_{0}\right)=y_{0}
\end{gathered}
$$

