(16) 1. Use the Residue Theorem to compute $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^2}$.

Comment `Maple` reports that the answer is $\dfrac{\pi}{2}$.

You must show details of any estimates required to apply the Residue Theorem to earn full credit here, in addition to the computation of any residues necessary.

(16) 2. Find $R > 0$ so that all roots of the polynomial $P(z) = z^5 + 12z^4 - (1+i)z^2 + 9z - 3$ are inside the circle $|z| = R$.

Comment You are not asked to find the "best possible" R. Even an extremely large value of R is acceptable but you must explain why your assertion is true (with *all roots* inside $|z| = R$) to earn full credit here.

(16) 3. Use the Residue Theorem to compute $\displaystyle\int_0^{2\pi} \frac{d\theta}{5 + 3\cos\theta}$.

Comment `Maple` reports that the answer is $\dfrac{\pi}{2}$.

(14) 4. Suppose

$$Q(z) = \frac{(e^z - 1)^2}{z^4}.$$

Identify as precisely as possible the type of the isolated singularity at 0 of $Q(z)$: is it removable, a pole, or essential? If it is a pole, find the order of the pole.
Find the first two non-zero terms of the Laurent series of $Q(z)$ at 0.
Find the residue of $Q(z)$ at 0.

Type of singularity ________________

First two terms ________________________

Residue at 0 __________

(12) 5. Suppose $f(z)$ is an entire function (analytic in the whole plane) and that $|f(z)| \le |e^z|$ for all complex numbers z. Show that there must be a complex number C with $|C| \le 1$ so that $f(z) = Ce^z$ for all z.

Hint Divide and use a famous theorem.

(14) 6. a) Compute $\displaystyle\int_{|z|=1} \frac{e^z}{z}\,dz$ using any applicable theorem.

b) Use the answer given for part a) to find the exact values of $\displaystyle\int_0^{2\pi} e^{\cos\theta}\cos(\sin\theta)\,d\theta$ and of $\displaystyle\int_0^{2\pi} e^{\cos\theta}\sin(\sin\theta)\,d\theta$.

Comment The newest version of `Maple` I have at home doesn't seem to be able to compute these integrals exactly. It can compute *approximate values* though: the first integral is approximately $6.28318\,5307$ and the second integral is approximately $-.32411\,23157\cdot 10^{-15}$.

(12) 7. The following information is known about a function, $F(z)$:

i) $F(z)$ is defined and analytic for all $z \neq 0$.

ii) $F(i) = 3$.

iii) For all positive integers, n, $F(\frac{1}{n}) = 0$.

a) What kind of isolated singularity must $F(z)$ have at 0? Explain your answer.

b) What is the radius of convergence of the Taylor series expansion centered at $z = i$ of the function $F(z)$? Explain your answer.

Second Exam for Math 403, section 1

April 22, 2002

NAME ______________________________

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes or calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	16	
2	16	
3	16	
4	14	
5	12	
6	14	
7	12	
Total Points Earned:		