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$$\iiint x^2 + y^2 dV$$

$$dV = \rho \sin \phi \, d\rho \, d\phi \, d\Theta$$

To turn into spherical coordinates,

$$f(x,y,z) = x^2 + y^2 = (\rho \cos \Theta \sin \phi)^2 + (\rho \sin \Theta \sin \phi)^2$$

$$= (\rho \sin \phi)^2 (\cos^2 \Theta + \sin^2 \Theta) = (\rho \sin \phi)^2$$

Since we are integrating the unit ball,  $0 \leq r \leq 1$  and  $r = \rho$

Also, we are integrating only where  $x \geq 0$  so  $-\pi/2 \leq \Theta \leq \pi/2$

Lastly,  $0 \leq \phi \leq \pi$

$$\int_{\Theta = -\pi/2}^{\Theta = \pi/2} \int_{\phi=0}^{\phi=\pi} \int_{\rho=0}^{\rho=1} (\rho \sin \phi)^2 * \rho \sin \phi \, d\rho \, d\phi \, d\Theta$$

$$\int_{\rho=0}^{\rho=1} \rho^3 \sin^3 \phi \, d\rho = \rho^4/4 \sin^3 \phi \text{ from } 0 \text{ to } 1 = 1/4 \sin^3 \phi$$

$$1/4 \int_{\phi=0}^{\phi=\pi} \sin^3 \phi \, d\phi = 1/4 \int_{\phi=0}^{\phi=\pi} \sin \phi (\sin^2 \phi) \, d\phi$$

$$= 1/4 \int_{\phi=0}^{\phi=\pi} \sin \phi (1 - \cos^2 \phi) \, d\phi \quad \text{let } u = \cos \phi \, dx = du / -\sin \phi$$

$$= -1/4 \int_{\phi=0}^{\phi=\pi} (1 - u^2) du = -1/4 (\cos \phi - \cos^3 \phi / 3) \text{ from } 0 \text{ to } \pi$$

$$= -1/4(-1 + 1/3) + 1/4(1 - 1/3) = 1/3$$

And then integrating the last integral,  $= 1/3 \Theta$  from  $-\pi/2$  to  $\pi/2$ , the answer becomes  $\pi/3$