(12) 1. Suppose that $f(x) = x^{3/2}$.

a) Find the third degree Taylor polynomial, $T_3(x)$, centered at $c = 4$ for $f(x)$.

b) Suppose $T_3(x)$ is the polynomial found in a). Use Taylor's inequality (the **Error Bound**) to find an overestimate for $|f(x) - T_3(x)|$ on the interval $[2, 6]$. Your answer should be an explicit number valid for every x on this interval.

Comment You are *not* asked to find the "best possible" estimate, only to find a convenient estimate which satisfies the requirement and to support your assertion. You need *not* simplify your answer!

(9) 2. a) Suppose that $f(x) = Ce^{(x^2)} + x$ (here C is an undetermined constant). Verify that $f(x)$ is a solution of the differential equation $y' = 2xy - 2x^2 + 1$.

b) Find a solution of the differential equation $y' = 2xy - 2x^2 + 1$ which passes through the point $(2, 3)$.

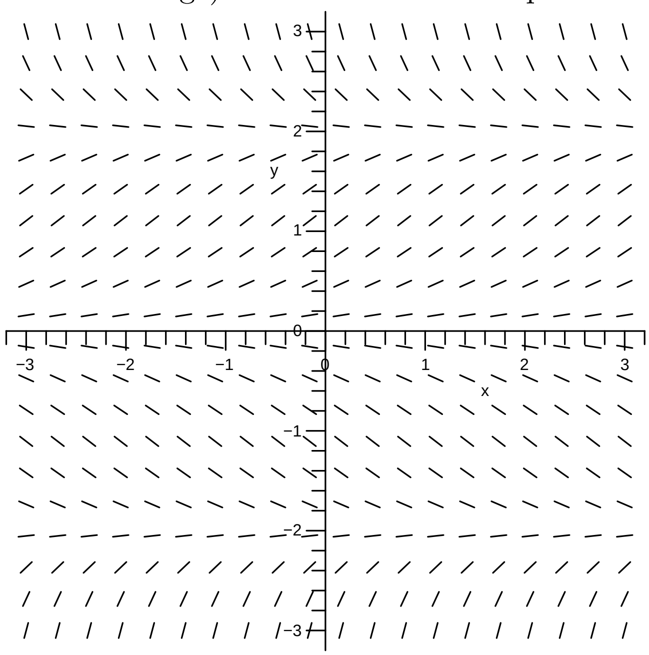
(11) 3. This problem is about the differential equation $y' = y\left(1 - \frac{1}{4}y^2\right)$.

a) Find the equilibrium solutions (where y doesn't change) for this differential equation.

b) To the right is a direction field for this equation. Sketch solution curves on this graph through the points below and find the indicated limits:

- $(0, 1)$. Label this curve **A**. On curve **A**, $\lim_{x\to-\infty} y(x) =$ ____ and $\lim_{x\to+\infty} y(x) =$ ____.

- $(0, -1)$. Label this curve **B**. On curve **B**, $\lim_{x\to-\infty} y(x) =$ ____ and $\lim_{x\to+\infty} y(x) =$ ____.



(12) 4. The infinite series $\sum_{n=1}^{\infty} \frac{3}{2\sqrt{n} + 4^n}$ converges and its sum, to an accuracy of .001, is .719.

n	4^n
1	4
2	16
3	64
4	256
5	1,024
6	4,096
7	16,384
8	65,536
9	262,144
10	1,048,576

Find a positive integer N so that the partial sum, $S_N = \sum_{n=1}^{N} \frac{3}{2\sqrt{n} + 4^n}$, has a value within .001 of the sum of the whole series. Explain your reasoning.

Comment You are *not* asked to find the "best possible" N, only to find a convenient N which satisfies the requirement and to support your assertion.

(12) 5. The infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5n^3+\sqrt{n}}$ converges and its sum, to an accuracy of .001, is .205.

Find N so that the partial sum, $S_N = \displaystyle\sum_{n=1}^{N} \frac{1}{5n^3+\sqrt{n}}$, has a value within .001 of the sum of the whole series. Explain your reasoning.

Comment You are *not* asked to find the "best possible" N, only to find a convenient N which satisfies the requirement and to <u>support your assertion</u>.

(12) 6. a) Suppose the sequence $\{a_n\}$ is defined by $a_n = (7n+3)^{5/n}$. Find the exact value of the limit of this sequence.

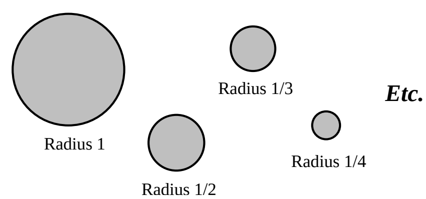
b) It is known that the sequence $\sqrt{3},\ \sqrt{3+\sqrt{3}}, \sqrt{3+\sqrt{3+\sqrt{3}}}, \sqrt{3+\sqrt{3+\sqrt{3+\sqrt{3}}}}, \ldots$ converges. Find the exact value of the limit of this sequence.

Hint Find a formula relating one term of the sequence to the next term. Then take limits.

(10) 7. There is an infinite sequence of circles which do not overlap and which have radius 1, $\frac{1}{2}$, $\frac{1}{3}$, ... as shown. (The n^{th} circle has radius $\frac{1}{n}$.)

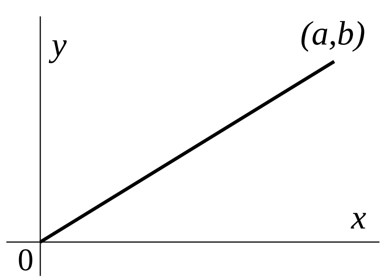
a) Is the total area inside all of the circles finite? (You are *not* asked to find the total!)

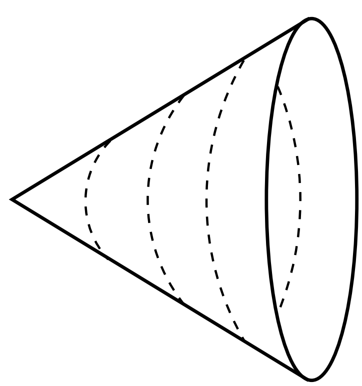
b) Is the total circumference of all of the circles finite? (You are *not* asked to find the total!)



(10) 8. Does the infinite series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(n!)^2}{(2n)!}$ converge or diverge?

(12) 9. Suppose a and b are unknown positive numbers. When the line segment connecting the origin to the point (a,b) is revolved about the x-axis, the result is the slanted surface of a cone. The area of this surface is $\pi b\sqrt{a^2+b^2}$. Verify this formula using calculus.





A **A**

Second Exam for Math 152

Sections 1, 2, 3, 6, 7, 8, and 9

APRIL 20, 2009

NAME ____

SECTION __________

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.

No texts, notes, or calculators other than the formula sheet may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	12	
2	9	
3	11	
4	12	
5	12	
6	12	
7	10	
8	10	
9	12	
Total Points Earned:		

Find exact values of standard functions such as e^0 and $\sin\left(\frac{\pi}{2}\right)$.
Otherwise do NOT "simplify" your numerical answers!

A **A**