(10) 1. Show that the surface area obtained when the curve $y = \sqrt{x}$ from $x = 0$ to $x = 2$ is revolved around the x-axis is $\frac{13}{3}\pi$.

(10) 2. Assume $m > 0$. Verify that the improper integral $\displaystyle\int_0^\infty xe^{-mx}dx$ converges and that its value is $\dfrac{1}{m^2}$.

Comment Be sure to explain why any special method used to compute limits is applicable.

(10) 3. a) Does the sequence defined by the formula $a_n = (7n^3 + 5)^{(2/n)}$ converge? If it does, find its limit. If it does not, explain why.

b) Does the sequence defined by the conditions $\begin{cases} b_1 = 1 \\ b_{n+1} = b_n + \frac{1}{b_n} & \text{if } n \geq 1 \end{cases}$ converge? If it does, find its limit. If it does not, explain why.

(8) 4. Bruno and Igor are again sharing a loaf of bread. Bruno, now hungrier and more ferocious, eats two-thirds of the loaf, then Igor eats eats half of what remains, then Bruno eats two-thirds of what remains, then Igor eats half of what remains, and so on. How much of the loaf will each student eat?

Comment *Please* give an answer that makes sense, with at least a little bit of supporting evidence. Thank you in advance.

(10) 5. I know that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5^n + 3^n} \approx .162$ with error $(\pm)$ less than .001. Find N so the partial sum $\displaystyle\sum_{n=1}^{N} \frac{1}{5^n + 3^n}$ is guaranteed to be within $.001 = \frac{1}{1{,}000}$ of the sum of the infinite series.

Comment You are *not* asked to find the "best possible" N, only to find a convenient N which satisfies the requirement and to support your assertion.

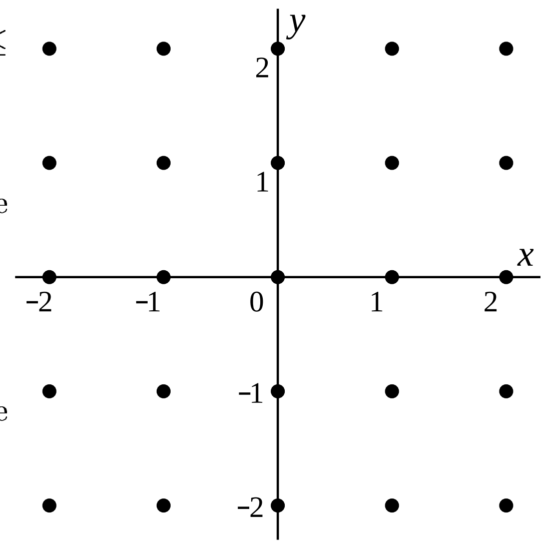
n	3^n
1	3
2	9
3	27
4	81
5	243
6	729
7	2,187

n	5^n
1	5
2	25
3	125
4	625
5	3,125
6	15,265

(8) 6. Solve the initial value problem: $y^2\dfrac{dy}{dx} = x^{-3}$ and $y(2) = 0$. In the answer express y explicitly as a function of x.

Section 9.1, exercise 32

(10) 7. a) Sketch the slope field of $\frac{dy}{dx} = xy$ for $-2 \le x \le 2$ and $-2 \le y \le 2$ at the indicated points.



b) Based on the sketch, determine $\lim_{x\to\infty} y(x)$, where $y(x)$ is a solution with $y(0) > 0$.

The limit is __________

c) Based on the sketch, determine $\lim_{x\to\infty} y(x)$, where $y(x)$ is a solution with $y(0) < 0$.

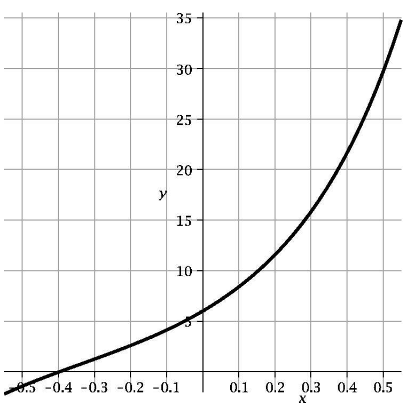
The limit is __________

Section 9.3, exercise 7

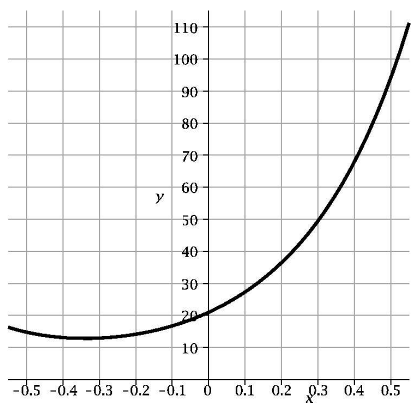
(12) 8. Suppose $f(x) = e^{x^2+\sin x}$. Here are values of f and some of its derivatives at 0:

$$f(0) = 1;\ \ f'(0) = 1;\ \ f''(0) = 3;\ \ f^{(3)}(0) = 6;\ \ f^{(4)}(0) = 21;\ \ f^{(5)}(0) = 52\,.$$

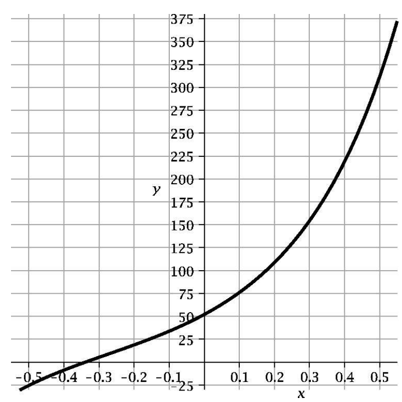
Below are graphs of $f^{(3)}(x)$, $f^{(4)}(x)$, and $f^{(5)}(x)$ on the interval $[-.5, .5]$.



Graph of $f^{(3)}(x)$ on $[-.5, .5]$



Graph of $f^{(4)}(x)$ on $[-.5, .5]$



Graph of $f^{(5)}(x)$ on $[-.5, .5]$

Assume this information is correct. No additional computation of the values of f or any of its derivatives is needed for this problem.

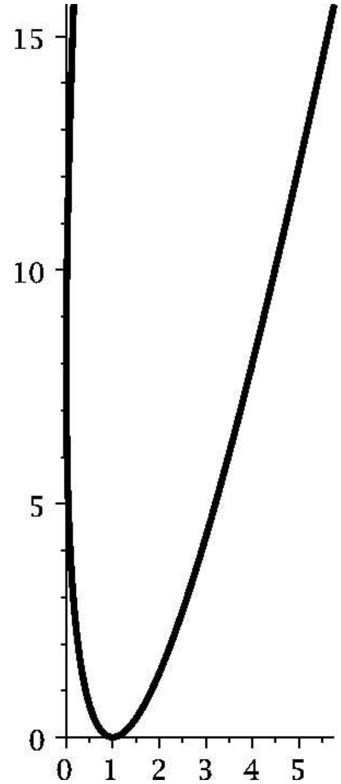
n	$n!$
0	1
1	1
2	2
3	6
4	24
5	120

a) What is the second degree Taylor polynomial centered at 0 of f? *Do no unnecessary arithmetic!*

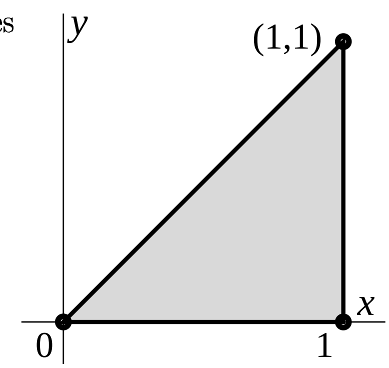
n	4^n
1	4
2	16
3	64
4	256
5	1,024

b) Find a polynomial $P(x)$ so that $|P(x) - f(x)| < .01$ for all x in the interval $\left[-\frac{1}{4}, \frac{1}{4}\right]$. You should write the polynomial and explain why the error is less than $.01 = \frac{1}{100}$.

(10) 9. Part of the graph of the parametric curve $\begin{cases} x = t^2 \\ y = 8(1-t)^2 \end{cases}$ is shown to the right. Find the (x, y) coordinates of the point on the curve which is closest to the origin.



(12) 10. Suppose that T is the triangular region shown with vertices at $(0, 0)$, $(0, 1)$, and $(1, 1)$.



a) Describe T using polar coordinates.

b) Compute the area of T using polar coordinates.

Second Exam for Math 152H

November 19, 2009

NAME ______________________________

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.

No texts, notes, or calculators other than the formula sheet may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	10	
2	10	
3	10	
4	8	
5	10	
6	8	
7	10	
8	12	
9	10	
10	12	
Total Points Earned:		

Find exact values of standard functions such as e^0 and $\sin\left(\frac{\pi}{2}\right)$.
Otherwise do NOT "simplify" your numerical answers!