Please hand in a solution to **ONE** of these problems on Thursday, December 3. There will also be four textbook homework problems posted to hand in that day.

1. Suppose A and B are positive numbers. Decide whether the following sequences converge. If they converge, try to find their limits. Your answers may involve both numbers and their relationship.

a) $c_n = \sqrt[n]{A^n + B^n}$ b) $d_n = \sqrt[n]{A^n + B}$

Hint Experiment! Choose various values of A and B and compute the first five or ten terms of each sequence. Then verify your guesses in general with algebra and calculus. The "experiments" need not be submitted but your general analysis should be.

2 a) Enter the number 5 in a calculator showing 10 decimal digits after the decimal point. Press the square root button 20 times. The result will be **1.00000 15348**. Subtract 1 and multiply by 1,048,576 to get **1.60943 91475** *but* the same calculator will declare that $\ln 5$ is **1.60943 79124**. Since 1,048,576 is 2^{20}, this is *not* a concidence. Explain.

b) Given a positive number, x, outline a strategy for computing $\ln x$ only with the arithmetic operations $(+, \times, -, /)$ and square root $(\sqrt{})$. Your strategy should involve asserting (and verifying) that a certain sequence which can be easily computed with the listed operations always converges to $\ln x$.

3. Consider an infinite series of the form

$$\pm 3 \pm 1 \pm \frac{1}{3} \pm \frac{1}{9} \pm \frac{1}{27} \pm \cdots \pm \frac{1}{3^n} \pm \cdots.$$

The numbers 3, 1, etc., are given but *you* will decide what the signs should be.

a) Can you choose the signs to make the series diverge?

b) Can you choose the signs to make the series sum to 3.5?

c) Can you choose the signs to make the series sum to 2.25?

In each case, if your answer is "Yes", then specify how to choose the signs; if your answer is "No", then explain.

4. Let $f(x) = (1 + x + x^2)\sin(x^3)$.

a) Find the first ten terms of the Maclaurin series of f (the Taylor series of f centered at 0). Use series that you know already!

b) What is $f^{(10)}(0)$?

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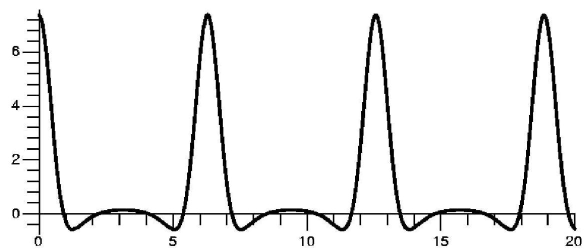
5. Suppose $f(x) = \sum_{n=0}^{\infty} \frac{2^n \cos(nx)}{n!}$. This series is not a power series. Shown to the right is a graph of the partial sum $s_{100}(x) = \sum_{n=0}^{100} \frac{2^n \cos(nx)}{n!}$ for $0 \le x \le 20$.



a) Verify that the series defining $f(x)$ converges for all x.

b) Is the apparent periodicity of the function $f(x)$ actually correct? If yes, explain why. Your explanation should include use of the term "periodic function" and explain why the defining condition is or is not satisfied.

c) Verify that the actual graph of the function is always within .01 of the graph shown. That is, if x is any real number, then $|f(x) - s_{100}(x)| < .01$.

Possibly useful numbers $2^{100} \approx 1.27 \cdot 10^{30}$ and $2^{101} \approx 1.54 \cdot 10^{30}$. Also, $100! \approx 9.33 \cdot 10^{157}$ and $101! \approx 9.43 \cdot 10^{159}$.

6. a) Is there a power series whose interval of convergence is the interval $(0, 1]$, that is, the interval defined by $0 < x \le 1$? Give an explicit series or explain why there is no such series.

b) Is there a power series whose interval of convergence is the interval $(0, \infty)$, that is, the interval defined by $0 < x$? Give an explicit series or explain why there is no such series.

7. One of the Bessel functions used to describe the vibration of a circular plate is defined by this infinite series: $J(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$.

a) Show that this series converges absolutely for all values of x.

b) Explain briefly why the result of a) implies that the series converges for all x.

c) Here are individual terms of the series for two values of x and for some values of n.

$\frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$	$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
$x = 1$	1	$-\frac{1}{4}$	$\frac{1}{64}$	$-\frac{1}{2{,}304}$	$\frac{1}{147{,}456}$	$-\frac{1}{14{,}745{,}600}$
$x = 4$	1	-4	4	$-\frac{16}{9}$	$\frac{4}{9}$	$-\frac{16}{225}$

Use entries of this table and facts about the series to explain why $J(1)$ must be positive and $J(4)$ must be negative.

Hint Select an N for each x and split the sum: $\sum_{n=0}^{\infty} = \sum_{n=0}^{N} + \sum_{n=N+1}^{\infty}$. Evaluate the finite sum explicitly and estimate the infinite tail $\sum_{n=N+1}^{\infty}$.