1. Sketch the three-sided region in the first quadrant bounded by the y-axis and the two curves $y = \tan x$ and $y = \sec x$. Compute the area of this region.

2. A computer program reports the following:

$$\int_0^1 \frac{x}{x+1}\,dx = 1 - \ln 2\,; \qquad \int_0^\infty \frac{t}{(2t+1)(t+1)^2}\,dt = 1 - \ln 2\,.$$

Verify that the two integrals are equal. Notice that you are *not* asked to evaluate these definite integrals, only to explain why the values are equal.

Hint Find the antiderivatives and compute both integrals: a very direct method.

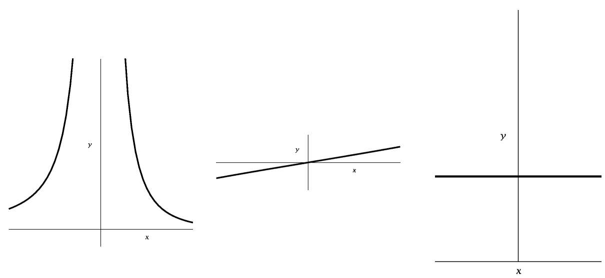
(Hint)^2 Change one integral into the other: x goes from 0 to 1 and t, from 0 to ∞ – everything involved is a rational function, so make the change from x to t with a simple rational function. After you find a suitable change of variables, how does dx change to dt?

3.* Find the limits for the following indeterminate forms of the type "$\infty - \infty$".

a) $\lim_{x\to 0} \frac{1}{\sin x} - \frac{1}{x}$.

b) $\lim_{x\to 0} \frac{1}{x^2} - \frac{1}{x}$.

c) $\lim_{x\to 0} \frac{1+x}{x} - \frac{1-x}{x}$.



4. Suppose that a is a positive constant and that R is the region bounded above by $y = 1/x^a$, below by $y = 0$, and on the left by the line $x = 1$.

a) Sketch the curves $y = 1/x^a$ for $a = .5$, 1 and 2. Which of these is closest to the x-axis?

b) For which positive numbers a do you get a convergent integral when you attempt to calculate the area of R?

c) Same as b), but for the volume of the solid obtained by rotating R around the x-axis.

d) Same as c), but for the volume of the solid obtained by rotating R around the y-axis.

5. The curve $y = e^{-x}$, $x \geq 0$, is revolved about the x-axis. Does the resulting surface have finite or infinite area? (Remember that you can sometimes decide whether an improper integral converges without calculating it exactly.)

* For those to whom L'Hôpital's Rule is new.