

1.* Suppose $F_A(x) = A^2(1 - x)x^A$. Here A is a positive real number .

a) Sketch $F_1(x)$, $F_{10}(x)$, and $F_{100}(x)$ for x in $[0, 1]$.

b) Suppose $g(A) = \int_0^1 F_A(x) dx$. Compute $g(A)$. What is $\lim_{A \rightarrow \infty} g(A)$?

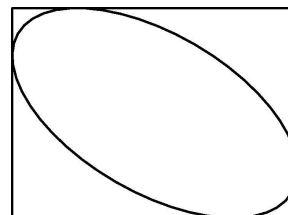
c) Consider x in $[0, 1]$ only and let $H(x) = \lim_{A \rightarrow \infty} F_A(x)$. Compute $H(x)$ and $\int_0^1 H(x) dx$.

d) Parts b) and c) compare $\lim_{A \rightarrow \infty} \int_0^1 F_A(x) dx$ and $\int_0^1 \lim_{A \rightarrow \infty} F_A(x) dx$. Do the pictures in part a) help you understand the results?

“In mathematics you don’t understand things, you just get used to them.” – John von Neumann

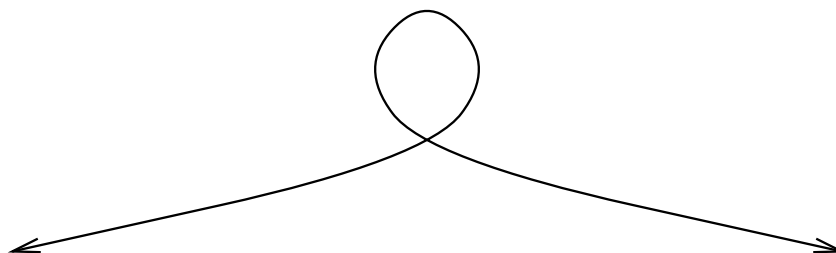
2. The parametric curve $\begin{cases} x = \sqrt{3} \sin t + \frac{1}{2} \cos t \\ y = -\sin t + \frac{\sqrt{3}}{2} \cos t \end{cases}$ is a tilted ellipse

whose graph is shown to the right. What are the dimensions and location of the box containing the ellipse?

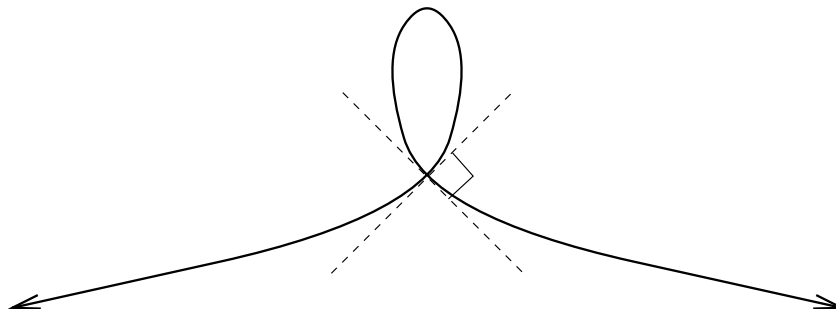


Note The sides of the box are vertical and horizontal and also are tangent to the ellipse. The box is called a “bounding box”.

3. If A is a positive number, the parametric curve $\begin{cases} x = t^3 - t \\ y = \frac{A}{1 + t^2} \end{cases}$ looks like:



The “self-intersection” of the curve is perpendicular for one value of A . The picture is this:



Find that value of A .

OVER

* Integrals and limits don’t have to work well together. Indeed, they frequently do not. Finding a simple example to compute and show this is not totally easy. What’s here is something “easy”, maybe.

4. The thread length for a simple spool of cotton thread is 25 yards. To celebrate Valentine's Day, purchase a spool of red thread and send it to your beloved with these instructions:

Unwind the thread and arrange it in the shape of a cardioid, $r = A(1 - \sin \theta)$.

The area of that cardioid represents how much I love you compared to the ordinary Valentine's Day card!

Compute the arclength of $r = A(1 - \sin \theta)$ and find A so that the length is 25 yards. Then compute the area inside that cardioid. Sketch the result.

Comment An opened standard greeting card seems to have area about 70 square inches, or about .054 square yards. On sale, a spool of thread costs about 25 cents. A card these days costs several dollars. Isn't the thread more cost-effective?

5. A window is in the shape of a 2' by 3' rectangle surmounted by a semicircle. Describe the boundary of the window as $r = f(\theta)$ in a polar coordinate system whose location and orientation you specify. The $f(\theta)$ likely will need to be defined "piecewise".

Comment A calculus course traditionally has a problem concerning this sort of window, including the rather uncommon word "surmount" as part of the description. Tradition is honored here!

