These problems are Taylored for you!

1. A rational number is a quotient of two integers. Thus, $\frac{27}{894}$ is a rational number, and so is $\frac{17}{44}+\frac{90}{2{,}103}-\frac{1}{337}$ (remember, we don't need to "simplify" in this class – I believe that sums, products, etc. of rational numbers *are* rational numbers).

a) Find a rational number that is within 10^{-100} of $\sin(.4)$.

b) Find a rational number that is within 10^{-100} of $\sin(1.4)$.

c) Find a rational number that is within 10^{-100} of $e^{2.8}$.

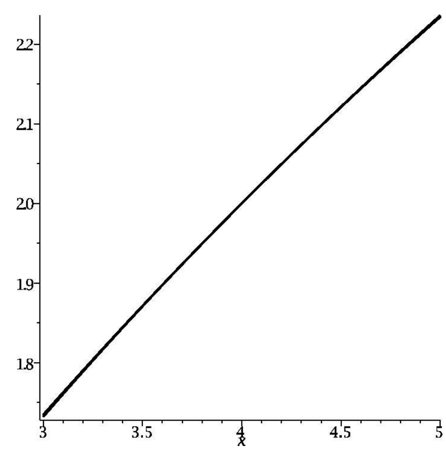
Comment You are *not* asked to exhibit, for example, the decimal approximation implied. You are only asked to show the rational numbers requested and give reasons why your approximations are correct. There are many possible correct answers to this problem.

2. a) Suppose $f(x)=5-6x+7x^2-8x^3$. Find the 100^{th} degree Taylor polynomial for f centered at $x=-2$.

b) Find (probably by explicit computation) the 5^{th} degree Taylor polynomial for $e^{(x^3)}$ centered at $x=0$.* Please try to guess, without explicit computation, the 10^{th} degree Taylor polynomial for $e^{(x^3)}$ centered at $x=0$.**

3. Fred *loves* polynomials with rational coefficients and only such polynomials. Suppose $f(x)=\sqrt{x}$. Find a polynomial $P(x)$ that Fred will adore so that, for any x is in the interval $[3,5]$, the difference between $P(x)$ and $f(x)$ is less than .01.

Hint The interval is $[3,5]$. What number is the *center* of that interval? And what is the function? And what is the topic we are probably investigating? To the right is a graph of $\sqrt{x}$ and a polynomial on $[3,5]$ (yes, two functions, even if you don't believe it). There are many polynomials which answer this question correctly. Please find one and explain why it is such a polynomial.



* Does this look like something to do with a Taylor polynomial for e^x

** Maybe it also is related to some Taylor polynomial for e^x and can be gotten from that.